THE SOLAR SYSTEM

BY

PERCIVAL LOWELL

THE SOLAR SYSTEM

Six Lectures

DELIVERED AT THE MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
IN DECEMBER, 1902

BY

PERCIVAL LOWELL

NON-RESIDENT PROFESSOR OF ASTRONOMY AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
AND DIRECTOR OF THE LOWELL OBSERVATORY, FLAGSTAFF, ARIZONA

BOSTON AND NEW YORK
HOUGHTON, MIFFLIN AND COMPANY
The Riverside Press, Cambridge
1903

Published May, 1903.

CONTENTS

CHAP.		PAGE
I.	OUR SOLAR SYSTEM	1
II.	MERCURY	27
III.	MARS	47
IV.	SATURN AND ITS SYSTEM	72
V.	JUPITER AND HIS COMETS	94
VI.	COSMOGONY	116

ELEMENTS OF THE SOLAR SYSTEM

TABLE		
I.	ORBITAL ELEMENTS	*facing* 134
II.	BODILY ELEMENTS	*facing* 134

ERRATA.

Page 23, line 8.

For $\frac{m}{a-d^2}$ *read* $\frac{2m}{(a-d)^2}$.

Page 74, line 18.

For Pierce *read* Peirce.

Page 104, second line under diagram.

For P *read* O.

Page 123, line 1.

After momentum *insert* projected at right angles to it.

LIST OF ILLUSTRATIONS

PAGE

Inner Planets 4
Outer Planets 5
Diagram 7
Meteor Streams 14
Conspicuous Comets 19
Mercury—Triad of Drawings 30
Libration in Longitude 31
Perturbative Action—Exemplifying the Origin of the Tides 36
Map of Mars 57
Drawings showing Identity between Canals and Rifts in the Polar Cap 63
Saturn's Rings 79
Position of Masses in Satellite Systems . . 85
Inclinations of Satellite Orbits to Primary's Equator 87
Jupiter's Family of Comets 100
Relative Orbits 104
Action of Jupiter 107
Comet Aphelia 114
Diagram 123
Faye's Laws of Attraction in Condensing Nebula 125
Successive Curves of Attraction in Condensing Nebula 126
Diagram 127
Axis Inclinations of the Major Planets . . 131

THE SOLAR SYSTEM

THE SOLAR SYSTEM

I

OUR SOLAR SYSTEM

Its position in space and in knowledge.

IN the long perspective of knowledge, which begins with the close at hand and stretches to the infinitely remote, the solar system marks a middle distance. Between the intimacy possible with objects on this Earth and the distant recognition of the universe of suns, it furnishes an acquaintanceship combining something of the interest of the one with the grandeur of the other.

Its constituents.

Our knowledge about the solar system has greatly increased during the last quarter of a century; and first in the recognition of what makes part of it. To our solar system we now know belongs every heavenly body we see except the fixed stars and the nebulæ. Not only are the Sun, Moon, and planets members of it, but meteors, shooting-stars, and comets we have found to be so, too. That all of these bodies are part and parcel of what the Sun controls, I shall first proceed to show you; for it is proper that we should recognize the members of the system before considering the system's constitution and the several characters of its constituents.

Obsolete views.

In many text-books you shall find it still stated that these flaming portents, the cometæ or long-haired stars, — for the ancients saw tresses where we prosaically see tails, — one of which, on the average, startles a generation into wonder, are visitors to us from other stars. So also we were taught that the strange stones that fall to us from the sky, and we call meteorites, were bits of some body from far interstellar space. Such knowledge belongs now to the history of science, not to science itself; for these bodies carry with them their badge of membership: it shows in the orbits they describe. So, when we pass through a comet's tail, or pick up a piece of meteoric iron, we now recognize that we have to do, not with a stranger, but with our own kith and kin. Man may gaze at matter beyond the solar system, but man has never yet *touched* it.

Path the proof of oneness.

Proof of community lies in the character of the paths. Planet and particle alike turn out to travel in ellipses, and ellipticity betrays association. How the orbit labels the occupant we shall see, on finding the paths the planets pursue and why they pursue them. The orbits of the planets are then the first point to consider.

Earth travels in an ellipse.

To begin with the Sun. Observation shows not only that the Sun changes its place in the heavens, but changes its size as well. To measurement through a smoked glass, it seems to contract in summer and expand in winter. Plotting the directions it successively takes in the form of a spider, and taking the legs inversely proportionate to the diameters at the times, we find an ellipse, in one of whose foci lies the Sun. The Earth, then, goes round the Sun in an ellipse.

So do the other planets.

To find the path of a planet, we first get its synodic period, or period with regard to the Sun. Then, from a sufficient number of observations of synodic periods to give their mean, we obtain the sidereal period, or period with reference to the stars.

By considering the angular motions, the two periods are easily seen to be connected by the following equation: —

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E};$$

Where E = the Earth's period;
S = the Planet's synodic period;
P = the Planet's sidereal period.

From two bearings separated by a sidereal period, we get a quadrilateral, of which, knowing parts enough to solve, we derive the planet's distance from the Sun at the moment. We now

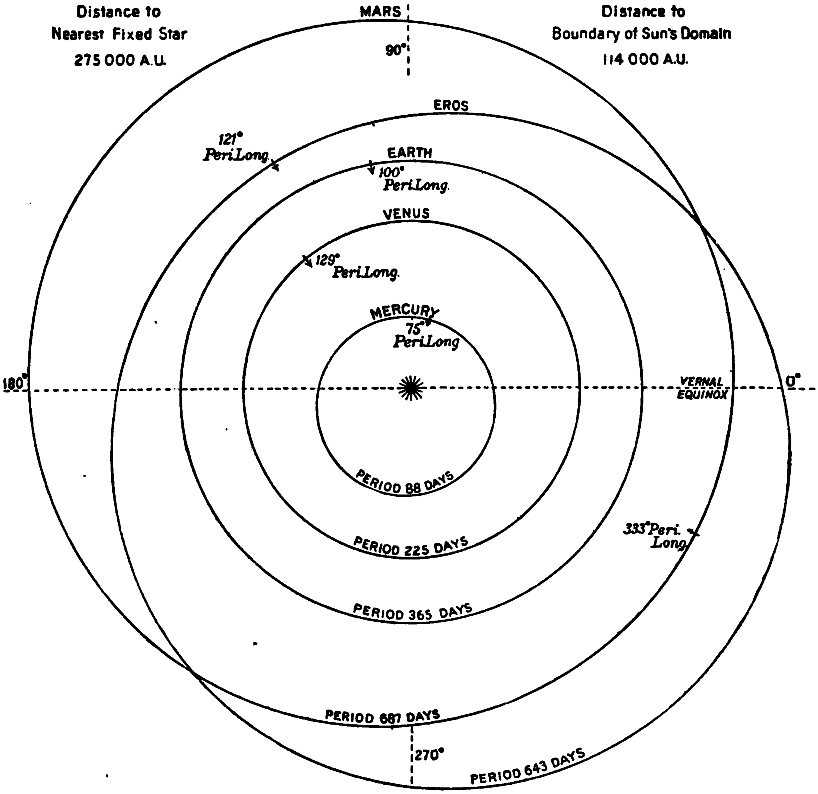


FIG. I. INNER PLANETS.

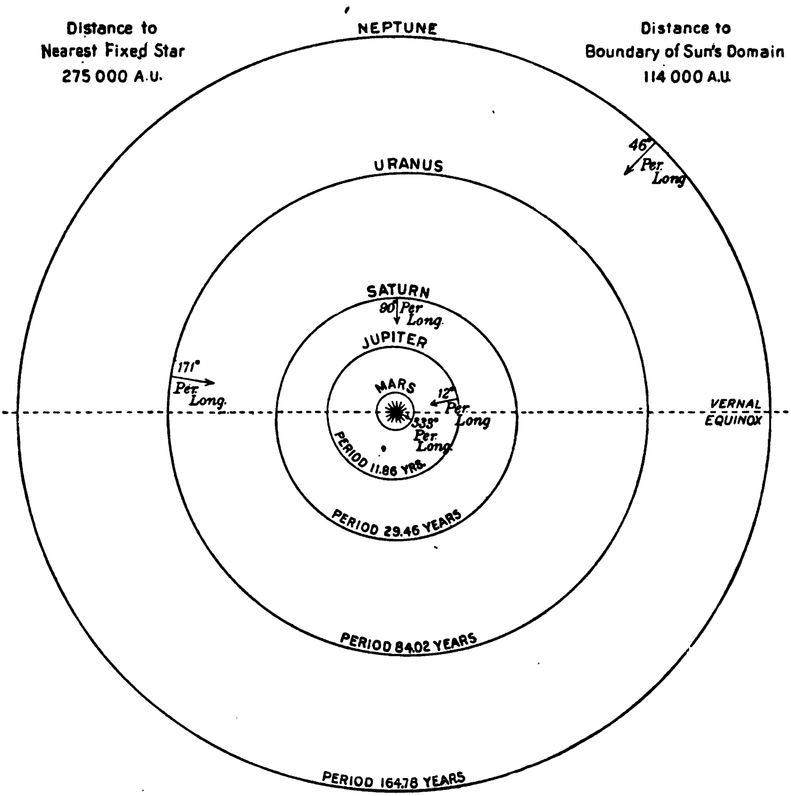


FIG. II. OUTER PLANETS.

have for the planet what we had for the Sun,—direction and distance at a given time. Dotting these data upon the apparent path, Kepler proved that the orbit of Mars was an ellipse. Mars was the first of the planets thus to have its orbit found; following it the others yielded similarly to the genius of the man. All the planets, then, move in ellipses about the Sun.

Thus we have obtained the accompanying plan of the system.

Kepler's laws.

Kepler discovered two more relations: first, that the radius vector of any planet swept over equal areas in equal times; and, second, that the cubes of the major axes of the orbits of any two planets were as the squares of their periodic times. The latter is not exactly true, but becomes so if we take the masses at work into account.

From these three "laws," Newton showed that the force governing the motions of the planets was in each case directed to the Sun, and was as the inverse square of the distance from him. Reversely he showed that such being the law of gravitation, the orbits must all be conic sections.

Ellipses and hyperbolas.

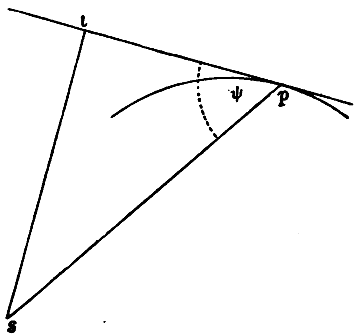
But conic sections are of two kinds, — ellipses or closed curves, and hyperbolas or curves that do not return into themselves. Clearly permanent members of a system must travel in the first of these two classes of curves, visitors only in the second. Here, then, we have an instant criterion for distinguishing bodies that belong to our system from those that visit it from without.

Which of the two orbits a body is pursuing may be determined either by actually finding the body's path or by finding the distance of the body from the Sun and its speed at the moment. For an interesting equation connects the speed with the distance, giving the major axis of the orbit, upon which alone the class of curve depends. This equation is

$$v^2 = \mu\left(\frac{2}{r} \mp \frac{1}{a}\right),$$

in which the — sign betokens the ellipse, the + sign the hyperbola.

Suppose a body at p moving along the curve whose tangent is pt with acceleration f always directed to s. Then $\dot{v}$, the resolved part of the acceleration along the tangent, is $f \cos \psi$.



The resolved part of the velocity v along sp is $\dot{r}$,

and $$\dot{r} = v \cos \psi;$$

whence $$f\dot{r} = v\dot{v} = \frac{1}{2} dv^2.$$

If $$f = \frac{\mu}{r^2}$$

$$\frac{1}{2} v^2 = \mu \left(\frac{1}{r}\right) + c.$$

c can be determined from the actual velocity at some point in the orbit (at the end of the minor axis, for instance, in the ellipse), and from this we can find that

$$v^2 = \mu \left(\frac{2}{r} \mp \frac{1}{a}\right)$$

where a is the semi-major axis of the curve, the upper sign referring to the ellipse, the lower to the hyperbola.

The velocity in the hyperbola thus exceeds that in the ellipse, and the dividing line between the two classes of curves is clearly when the second term is zero.

Consequently

$$v^2 = \frac{2\mu}{r}$$

is the velocity which at any given distance r separates the bodies moving in ellipses from those moving in hyperbolas, the sheep from the goats.

$$v^2 = \frac{2\mu}{r}$$

is called the parabolic velocity, but the student should be careful to remember that the parabola is a mathematical conception, not a physical fact.

It is a conceptual dividing line between ellipses and hyperbolas, the paling between the sheep and the goats.

Now the planets all move in ellipses. They are therefore under the Sun's control and form part of his system.

Meteors.

Occasionally stones fall out of the sky on to the earth. Suddenly a flash occurs overhead, a detonation follows, and then if the observer be near enough, a mass of stone or iron is seen to bury itself in the ground. This is a meteorite, aerolite, or bolide, a far wanderer come at last to rest.

The flash, the report, and the fused exterior of the mass found are due to the meteor's striking against our air. The bodies enter the upper atmosphere at speeds of from ten to forty miles a second, and such speeds are equivalent to immersing them in a blow-pipe flame of a temperature of many thousands of degrees. For the temperature of a gas is as the mean velocity-square of its molecules, and the rush of the meteor produces the same effect as if the molecules of the air were moving and the air therefore very hot.

Previously cold.

Its outward condition is a consequence of the last stage in its journey, but its inner state at times continues to bear witness to a previous condition. If the mass be large, time does not suffice to fuse more than its exterior, and the interior retains the cold of interplanetary space. As Young tells us, one of the fragments of the Dhurmsala meteorite in India was found in moist earth, half an hour or so after its fall, *coated with ice!*

Their orbits short ellipses.

But their speed is the real tell-tale upon their past. An ingenious investigation by the late Professor Newton, whose specialty was these very things, proved that ninety per cent., and probably all of the meteorites for which we have sufficient data, were traveling, before their encounter with the earth, in orbits not parabolic, but elliptic, like those of the short-period comets, and were moving direct. They come to us, therefore, not from the stars, but from the Sun's own domain. They, too, then are members of the system.

Their origin.

Most interesting is their constitution in its bearing upon their origin. Some are stone, some iron — meteoric iron joined with nickel. Now the iron meteorites are saturated with occluded gases, which can be extracted from them by suitable processes, and which cannot have been occluded originally except in the molten interior of a sun, intense heat and excessive pressure being necessary; and as they are now ungathered in remnants of our own once nebulous mass, they must betray what that nebulous mass was to begin with; for in their subsequent history there has been nothing to make them what they are. They cannot have come from our present sun, since it became a sun, as their orbits conclusively show. They must have come from the sun our system had before the catastrophe, which caused the nebula which caused our Sun, occurred. They antedate the creation of the nebula itself which our nebular hypothesis posits as the beginning of things. They are old with an age which staggers imagination; older in cycles of evolution, if not in years, than anything we see in the countless spangles of a winter's night in the blue-black firmament of sky. Before the silent tale they tell, history shrinks into yesterday, the Earth's career into the day before, and the evolving of the solar system itself into modernity. Through that strange Widmannstättian fretwork that marks their surface like the lacing of frost-work on a window-pane, we seem to be gazing past the iron bars into the immensity, not of space alone, but of eternity.

Shooting-stars.

Next to meteors, and doubtless close to them in kind, come shooting-stars. Superficial distinctions have caused them to be classed apart, but in all likelihood size alone separates the two.

In the case of shooting-stars, we have the flash, the lingering scarf of light left when the body itself has eluded us, but no sound is heard, and nothing reaches the earth.

Meteor-streams.

The visitants come, too, in swarms. They have their times and seasons. Different nights of the year are consecrate to special flights; and the successive years bring back the same flights like birds that honk overhead at the same recurrent season of the year.

Such regularity has caused them to be noted and studied, and we have now a score of well-recognized congeries of shooting-stars or meteoric streams, known, for example, as the Leonids, the Perseids, the Andromedes. Each swarm has its radiant or perspective point from which all its members seem to come. From this radiant it derives its name, the Leonids seeming to come from a point in the constellation Leo, the Orionids from the constellation of Orion, and the Lyrids from the Lyre.

Their speeds.

Each of these swarms enters our atmosphere with cosmic speed, all the shooting-stars of one swarm traveling at the same rate; but each swarm has its own distinctive velocity. The Andromedes move relatively slowly, — eleven miles a second, — and are reddish. They overtake us; this accounts for their sluggishness, and their sluggishness explains their color. They are only red-hot. The Perseids move with medium velocity. They strike us on the quarter at twenty-five miles a second, and they are yellow. The Leonids, or November meteors par excellence, meet us head on at forty-three miles an ~~hour,~~ second their swiftness giving them a bluish-green tint or a white heat.

Their orbits elliptic.

To Professor Newton again we owe our first step to knowledge of them. After the shower of the Leonids in 1866, he determined, from all the observations upon them, five orbits which they might have pursued; and then Adams, of Neptunian fame, from the motion of their node, showed that only one of the five, an orbit with a period of thirty-three years, would satisfy the problem. Thus was explained the similar shower of 1833 and the yet earlier one of 1799, seen by Humboldt. We should have had them again in 1900, but that Jupiter probably interfered.

In the same way, the Andromedes prove to travel in an orbit whose period is thirteen years, and whose aphelion lies just outside the orbit of Jupiter. So, also, the Perseids pursue a closed orbit, but a much larger one, which takes them far beyond the orbit of Neptune.

Shortly after Newton and Adams had worked out the path of the November meteors, Schiapa-

Association of meteor-streams with comets.

relli attacked the orbit of the Perseids, or August meteors, and to the astonishment of the scientific world brought out the surprising fact that they

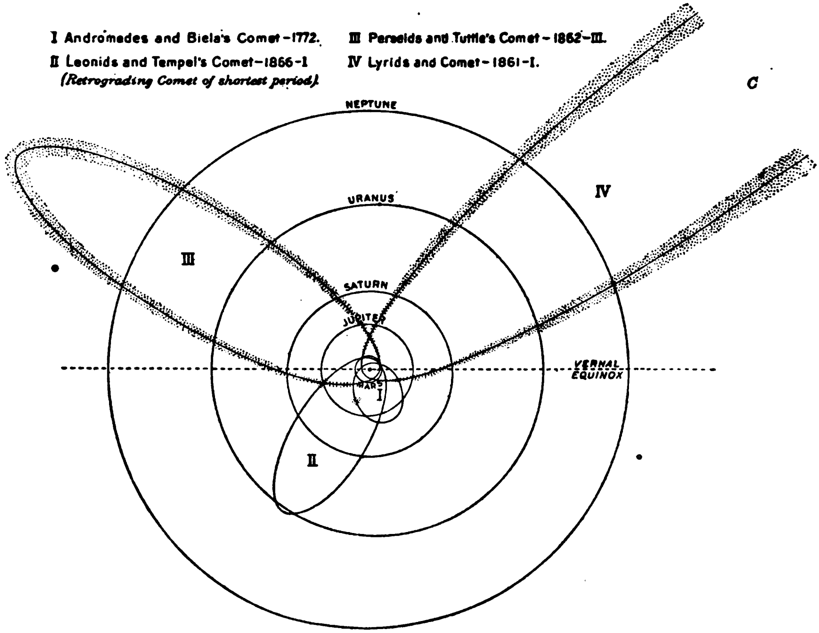


Fig. III. Meteor Streams.

traveled in an orbit substantially coincident with that of the great comet of 1862, known as Tuttle's comet (1862 III.). About the same time, Leverrier published his orbit of the Leonids, and nearly simultaneously Oppolzer, the great comet computer, published his of Tempel's comet of 1866 (1866 I.), and the two were found to be practically identical. Here were two identities which could hardly be the result of chance. Researches since have added to the number of such comet-meteor associations. Professor Herschel catalogues seventy-six; and four pairs — the Leonids and Tempel's comet, the Perseids and Tuttle's comet, the Andromedes and Biela's comet, and the Lyrids and the comet of 1861 (1861 I.) — are shown in the diagram on the opposite page.

Comets become meteor-streams.

Thus are comets and meteors connected. But we know more about their connection than this simple fact of association. We know that the one becomes the other, for we have seen the process of transformation take place practically under our very eyes. Biela's comet was for many returns a well-ordered member of Jupiter's comet-family, of which family we shall have more to say in the fifth chapter. Up to 1839 it had returned with due regularity and without incident. In 1846, it again appeared on time, but thereupon proceeded to do something very strange and then unheard-of. In mid-career it split. It was first seen on November 28, and presented the appearance of the usual comet. By December 19 it had become pear-shaped, and on January 13 it divided, the two halves at first separating, and thenceforth traveling side by side at a distance of one hundred and sixty thousand miles for the subsequent four months during which they continued visible. A bridge of light sometimes spanned the interval between them.

In 1852 the two returned. The distance between the pair had now increased to one million five hundred thousand miles, and they traveled thus during the time of their visibility.

Neither has ever been seen since; but in 1872, just when the Earth was passing the track of the lost heavenly twins, on November 27, occurred a brilliant star-shower. The German astronomer, Klinkerfues, was so impressed with the belief that this must be the remains of the comet, and that the comet itself, or what was left of it, would be seen exactly opposite the radiant, that he telegraphed at once to Pogson, the government astronomer at Madras, India: "Biela touched Earth November 27; search near Theta Centauri." Pogson looked. Clouds at first prevented, but on the third morning it was fair, and he saw in the predicted place a comet with a round head and a faint tail moving as it should have done. The next morning he observed it still better, and in its proper place. Oppolzer, by assuming the major axis, showed that this may have been Biela's comet.

Since then, other comets have been observed to split up, due to the action of the planets near which they chance to pass; and Callandreau has shown that the event ought not to be so very uncommon.

Meteor streams attendant upon planets.

Another point connected with these meteor streams must be noticed. Each of them is associated with the orbit of some particular planet. The planet in some sense shares with the Sun a control over the stream. It cannot cause the stream to circle round itself, but it can, and does, cause it to pay periodic obeisance to its might. The stream's perihelion remains at the Sun, but its aphelion becomes its periplaneta. It sweeps about the planet at the one end of its path somewhat as it sweeps round the Sun at the other.

The Andromedes are thus dependent on Jupiter, the Leonids on Uranus; while the Perseids and the Lyrids go out to meet the unknown planet which circles at a distance of about forty-five astronomical units from the Sun.

It may seem to you strange to speak thus confidently of what no mortal eye has seen, but the finger of the sign-board of phenomena points so clearly as to justify the definite article. The eye of analysis has already suspected the invisible.

Conspicuous comets.

In our identification of the members of our system we have thus got steadily farther and farther away. We began with the planets. Then we attacked the less evident and more erratic bodies, and we found that the nearest of them, the meteorites, were after all fellow-members, and circled quite near us, their orbits being comparable with, and possibly not alien to, the short-period comets.

Next we found the shooting-stars, the meteor streams, to be sun-controlled but traveling farther yet out into space, and connected with comets known to be periodic. We have now to take another step outward to the comets non-periodic, among which the most conspicuous of those visitants are numbered.

Non-periodic we may call them pending investigation. For their orbits are so vast that we know but vaguely what their major axes are.

Parabolic comets.

Some four hundred of these stars with tresses have been seen from the earliest times of which we have records to the present day. Not a year passes that several are not discovered, but conspicuous ones are not over-common. In the last forty years there has been but one of superlative mien, and that was twenty years ago. The present generation has no conception of what a comet worthy the name can be. One of my first recollec-

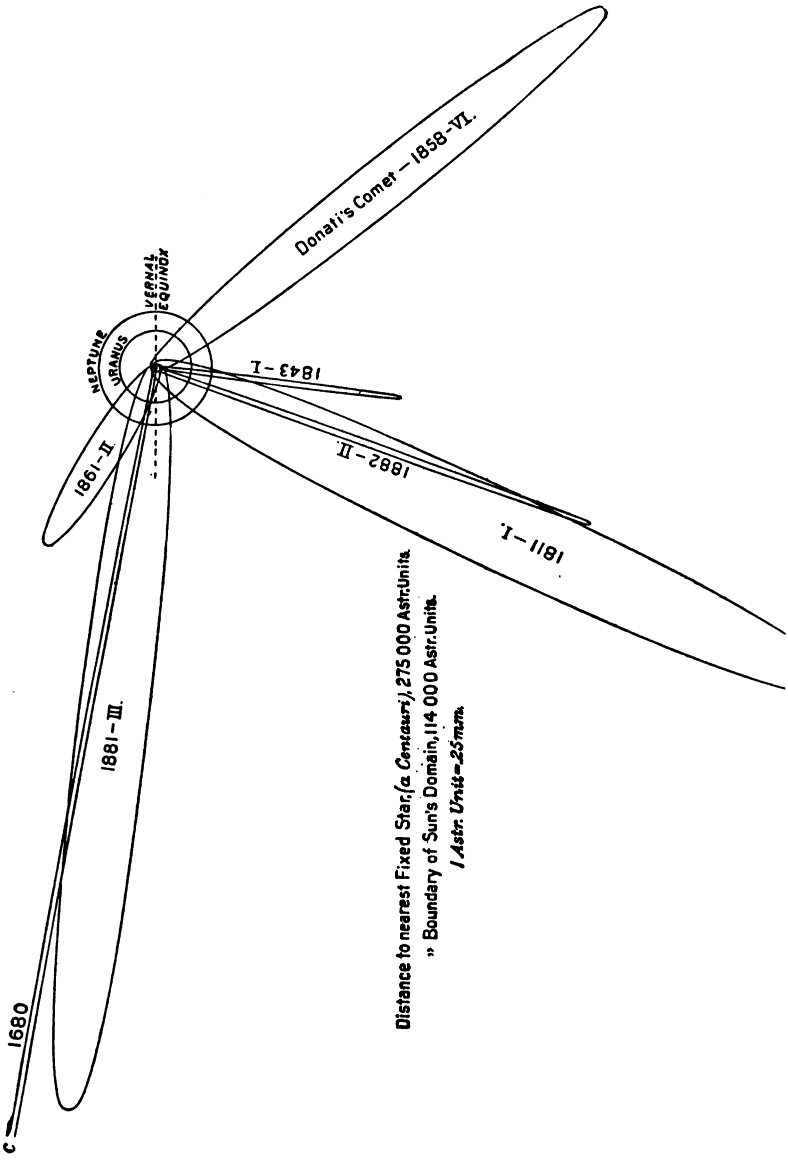


FIG. IV. CONSPICUOUS COMETS.

tions, if not my very first, is of such an one, and the memory of it has never been approached by any celestial phenomenon since. A total eclipse of the Sun is commonplace beside it. Of the four hundred up to now observed, the greater part move in orbits differing so little from the parabolic for the small fraction of their paths we are privileged to mark that to all intent they travel in parabolas. They lean, however, to the side of the ellipse. Most of them frankly do so, although so slightly that to determine their major axes to any degree of accuracy is not possible. Very few, three or four perhaps, hint at hyperbolas. Not one is such beyond question, however slightly. In my notes on Galle's catalogue, I find the following gloss at the end of the list: "There is not a single undisputed hyperbolic orbit; nor is there one in which the computed non-hyperbolic orbits are not in the majority."

Their orbits.

From this fact of a practical parabolicity of path many astronomers have argued the exterriterritoriality of these bodies, and early in the last century Laplace set himself the problem of finding the probability of hyperbolic to elliptic orbits on the theory that they all came to the Sun from stellar space. In spite of several mistakes in his work, first pointed out by Gauss, he reached a conclusion which is correct in quality: that the number of hyperbolic orbits to elliptic should be very small, less than one in the whole number already seen, on the tacit assumption that the Sun was at rest.

Conclusion as to connection with the solar system.

But the Sun is not at rest. It is traveling at the rate of eleven miles a second towards a point in the constellation Hercules, carrying its retinue with it; and this motion quite alters the result. Instead of a great preponderance of elliptic orbits, the solution shows in this case a large excess of hyperbolic ones. And in most of the orbits the hyperbolicity would be marked, not faint and doubtful. To Schiaparelli we owe the first suggestion of this fact, and, in 1895, to Fabry, of the observatory of Marseilles, a very elegant and conclusive memoir on the subject.[1]

In view of this we see that comets behave not as they would, did they come to us as visitors from other stars, but just as they should, considered as distant members of our own system. Comets, then, are also all co-members of the system.

The sun's domain.

That there is quite room enough within the Sun's paramount domain for their gigantic orbits becomes evident when we consider the distance to which that domain extends. Measured even

[1] *Annales de la Faculté des Sciences de Marseille.*

on the vast scale of our solar system, the gap which sunders it from the nearest fixed star is something enormous. Two hundred and seventy-five thousand times our distance from the sun is the space that divides us from the next sun, the star α Centauri. This distance is found by noting the shift in the star's position due to the extreme swing of the Earth in her orbit called the annual parallax. It is a very minute displacement at most, and requires perhaps the most delicate of all astronomical refinement to detect. Incidentally it affords conclusive evidence of itself that the Earth goes round the Sun, not the Sun round the Earth.

α Centauri.

Fortunately α Centauri, our nearest stellar neighbor, is a double star, a binary system, and thus of itself affords us information of the region over which it exercises control. Assuming that gravity acts there just as it does here, — any other possible assumption implies that the force depends on the orientation, which does not seem rational,[1] — we can deduce from the motion of the

[1] Binaries move in apparent ellipses. Parallel projection keeps an ellipse an ellipse and the centre the centre. From the general polar equation of a conic and the differential equation of the orbit,

$$f = h^2 u^2 \left(u + \frac{d^2 u}{d\theta^2}\right),$$

it appears that the only laws of force which do not depend on pair their united mass. It comes out twice that of our Sun. Now, as gravity is as $\frac{m}{d^2}$, we have, calling the whole distance from us to them *a*, the following quadratic to give us *d*, the boundary distance between the two domains, our Sun's and α Centauri's,

$$\frac{m}{a-d^2} = \frac{m}{d^2};$$

from which we find the dividing line between the Sun's domain and α Centauri's to be 114,000 astronomical units.

Neptune, the farthest known planet at present, is but thirty astronomical units away, or about $\frac{1}{4000}$ only of the distance to the limit of the Sun's domain. How nestled we all are under the Sun's protecting wing is evident. It is no wonder that the remotest comets seem almost infinitely distant at their aphelion, though part and parcel of the brood.

The several planets.

Coming back now from these chill outer confines of the Sun's territory to the inner family circle gathered about the hearth or focus of all these ellipses, occupied by the Sun, — for such is

θ, that is, on the orientation, are $f = cr$, which is negatived by the fact that no star has yet been found in the centre of the apparent ellipse, and $f = \frac{c_1}{r^2}$, which is thus the only law possible which is rational. It is thus ably put by Moulton (*Celestial Mechanics*, 1902).

the literal meaning of the word "focus," — we must note how the main bodies and yet smaller particles are severally ranged about it.

Terrestrial and major planets.

Humboldt divided the planets into two groups: the terrestrial planets and the major planets, and this classification one shall still find in many a text-book. But it has long since ceased to contain even a specious distinction. The so-called terrestrial planets differ among themselves quite as much as any of them do from the major planets. From our present knowledge it would be much nearer the mark to divide the eight into pairs, Mercury and Venus, the Earth and Mars, Jupiter and Saturn, Uranus and Neptune; yet even between the members of each pair are notable differences, to say nothing of the asteroids which throng the space betwixt Jupiter and Mars.

Of the differences, it will be the province of the succeeding chapters to speak; but before doing so, let us take a bird's-eye view of the whole.

Solar System a single-star one.

Our own Solar System has one characteristic, a general family trait, which distinguishes it from many that lie round about it in space; for we may not doubt that the stars are centres to systems of their own. We have not only analogy to guide us to this deduction, but we already have glints of evidence of the fact. Our system differs, however, from many of its neighbors in being a single-sun system. This is a very important and fundamental distinction. To begin with, it makes cosmic principles much easier to understand. We think celestial mechanics abstruse enough as they are, but ours are child's play to the complications which two suns, to say naught of three or four, would introduce into any system over which they jointly held sway. It is problems of this nature which Professor Darwin and other modern analysts are trying to unravel. Difficult as the conceptions are, it is a question whether life itself would not be quite as difficult, under such conditions. Take our nearest stellar neighbor, α Centauri, for instance, and consider what a planet circling round either or both of its suns would be called upon to undergo. Certainly our orderly succession of phenomena would be seriously disturbed to the consequent inconsequency of development upon its surface. Day and night would become meaningless terms, and organisms would have to put up with variations which make imagination stare.

For fashioning worlds like the terrestrial, a single-star system is, in general, a prerequisite.

This oneness is due to the system's original small moment of momentum. A minimum moment of momentum is caused by the centralization

Due to small initial moment of momentum.

of the mass; a maximum by its equal division into two or more. If we calculate the moment of momentum of the Solar System to-day, and compare it with that of any binary system, we shall find it in comparison almost vanishingly small.

The system, 61 Cygni, with only one fifth of its mass, has a moment of momentum two hundred and fifty times as great, and that of α Centauri, which has twice the mass, has two thousand times the moment.

This means that in the region of space, which made room to the solar nebula, the individual motions must have been either small or equally large in all directions, the negative motions almost exactly canceling out with the positive ones.

II

MERCURY

NEAREST to the Sun of all the bodies of the system, excepting only the swarm of particles which give us the Zodiacal Light, is Mercury.

Till very lately, we knew next to nothing about this planet. Its doings, as represented by its path, were well determined, but its self not at all. Part cause of this was its nearness to the Sun; a part, its being an inferior planet, and thus being but ill seen when most observable; for when at its greatest apparent distance from the Sun, — at one of its elongations, as it is called, — half of it alone is illuminated, and that half but poorly. Secondly, when it appears to the naked eye, and when in consequence it is generally looked for with the telescope, it is deep sunk in the vapors of the horizon, and the air through which it is seen is so tremulous that its disk, in consequence, is ill-defined. As this was supposed the best time for observation, the disk was deemed inscrutable.

Till lately very little known of it.

But the obvious is to be avoided. Acting upon this principle, Schiaparelli, in 1889, took a new departure by systematically observing Mercury by

Markings detected in 1889.

day. He was before long rewarded. Markings began to show themselves upon the little disk, difficult of detection, indeed, but still visible enough to enable him to be satisfied of their permanency; and then the markings disclosed of themselves a very singular fact. From the stability of their positions, it became evident that the planet rotated upon its axis in the same time that it revolved about the sun.

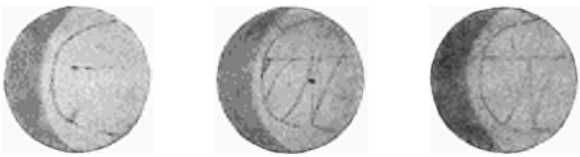
Rotation and revolution isochronous.

Let us consider a moment how it was the markings disclosed this fact. Suppose, for simplicity, a body revolving round its primary in a circle and made visible by the light received from it. Furthermore, suppose the revolving body to have markings upon it, and to rotate once upon its axis as it makes one revolution round its sun. Clearly it will always present the same face to the central attracting and illuminating body, and therefore the markings will maintain an invariable position with regard to the illuminated face. To an outsider, the planet, if inferior, will present the phases of the Moon. Unlike the Moon, however, the illumination will not sweep over an invariable face, but lighting and lighted will rotate together; for in the case of the Moon, we are the attracting, but not the illuminating, body; in the case of a planet, the Sun is both.

Flagstaff corroborates Schiaparelli.

Schiaparelli was the only one to see these markings till 1896, when the subject was taken up at Flagstaff. The planet was at the time coming out from inferior conjunction, and was at first no easy matter to find; for in relative visibility Mercury behaves like the Moon. Size of disk does not begin to compensate for phase, as calculation would lead one to expect; because obliquity of illumination greatly enfeebles its amount. The planet presented so faint a contrast with the sky that on one occasion an assistant, coming to look at it through the telescope, could not see it until its exact position was pointed out to him; and I always picked it up myself by trailing it across the field, an object in motion being much more evident than one at rest, as every hunter knows. Nor could I at first make much out of it; it was only a pretty little moon nearly lost in the vast blue sky. To my surprise, however, as it left elongation to return to the Sun, it grew brighter and brighter, and distinct dark markings came out upon its disk. The best views occurred when popular almanacs inform their readers: "Mercury invisible during the month." In the clear sky and steady air of Arizona and Mexico the markings were not especially difficult objects, though more difficult than the canals on Mars. They were narrow, irregular lines and very dark. They were not in the least like the markings on Mars. There were no large patches of shade on the one hand, nor fine, regular pencilings on the other. Its lines were fairly straight, but broken and of varying width. "Cracks" best explains their appearance, and probably their nature.

Their positions were unmoved, even after as much as five hours' interval.



Nov. 1—21h 16m. Nov. 2—0h 38m-45m. Nov. 2—2h 40m.

FIG. V.

TRIAD OF DRAWINGS, NOV. 1-2, 1896.

No shift in markings during 5h 24m.

Markings reveal libration in longitude.

As I continued to map them, I marked that while their relation to the terminator was unchanged by the hours, it was slowly shifting with the days. The lines were gradually passing over its edge, and it dawned on me what I was witnessing: the swaying, or libration, of the planet in longitude due to the eccentricity of the planet's orbit.

MEMOIRS AMERICAN ACADEMY, Vol. XII. PLATE XXXI.

Oct. 19 — 23h 45m.
$\iota = 102°$. $\omega = -4°$.

Oct. 24 — 23h 38m-
Oct. 25 — 0h 4m.
$\iota = 76°$. $\omega = -15°$.

Oct. 28 — 22h 49m-57m.
$\iota = 60°$. $\omega = -20°$.

Oct. 31 — 23h 18m-32m.
$\iota = 49°$. $\omega = -23°$.

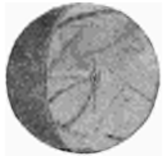
Nov. 8 — 22h 54m-23h 8m.
$\iota = 28°$. $\omega = -23$.

1896.

Feb. 5 — 23h 23m-37m.
$\iota = 102°$. $\omega = -22°$.

Feb. 14 — 21h 40m-22h 2m.
$\iota = 82°$. $\omega = -16°$.

Feb. 27 — 22h 19m-28m.
$\iota = 60°$. $\omega = 0°$.



Mar. 3 — 20h 35m-48m.
$\iota = 55°$. $\omega = +5°$.

Mar. 20 — 19h 56m-20h 14m.
$\iota = 28°$. $\omega = +23°$.

1897.

ι = the angle between Sun and Earth at Mercury.

ω = the angle of libration.

FIG. VI.

LIBRATION

DUE TO THE ECCENTRICITY OF THE PLANET'S ORBIT
MADE VISIBLE BY THE MARKINGS.

LOWELL OBSERVATORY, 1896–7.

All the Drawings are by Percival Lowell.

Cause of libration in longitude.

Libration in longitude is a necessary consequence of the planet's moving in a focal conic. The moment of rotation of a body of Mercury's mass is so great that it would take more than the Sun's might to suddenly alter it. The planet turns upon its axis, therefore, with a uniform spin. But its angular speed in its orbit is not uniform. Since the radius vector sweeps out equal areas in equal times, the angular velocity near perihelion exceeds that near aphelion. The revolution gains on the rotation here, and at the end of a certain time reaches its maximum; after which the rotation gains on the revolution, and the deficiency is made up again at aphelion.

Maximum point of libration in longitude.

To determine what the maximum is, and where, we have: that the mean angular velocity of revolution in the ellipse is the angular velocity of a body supposed to be describing a circle in the time occupied by the planet in the ellipse. The area of the ellipse being πab, and the period T, the areal velocity in the ellipse, which is constant, is

$$\frac{\pi a b}{T}.$$

This is the areal velocity in a circle of radius $\sqrt{ab}$ supposed described in the same time.

To find, therefore, the point on the ellipse where the radius has the value corresponding to the mean angular velocity, we must take the expression for r of the ellipse referred to its focus as a pole,

$$r = \frac{a(1 - e^2)}{1 + e \cos v},$$

and equate it to that of the circle supposed described about that focus with the length of radius $\sqrt{ab}$. This geometrically is the point of intersection of the two curves, since the value of r is common to both.

Consequently for the point sought

$$\sqrt{ab} = \frac{a(1 - e^2)}{1 + e \cos v}$$

whence, since $$b = a\sqrt{1 - e^2},$$

$$(1 - e^2)^{\frac{1}{4}} = \frac{(1 - e^2)}{1 + e \cos v}$$

and $$\cos v = \frac{(1 - e^2)^{\frac{3}{4}} - 1}{e}.$$

In the case of Mercury, $e = .205605$; v, the true anomaly of the point of maximum libration, is therefore 98° 55′.13.

But $$\frac{a - r}{ae} = \cos E,$$

where E is the eccentric anomaly; and $E - e \sin E = M$, where M is the mean anomaly; whence $v - M = \zeta$, which is the amount of the maximum libration, is 23° 40′ 38″.

The gain or loss of the rotation over the revolution is the same thing as the equation of the centre.

We have, then, in the libration, a most conclusive and interesting proof of the isochronism of rotation and revolution.

The next point to consider is what caused this

New branch of celestial mechanics.

isochronism. This question raises a wholly new set of problems in celestial mechanics from those in which celestial mechanicians were wont to engage. Until recently, mathematical astronomy dealt almost entirely with solids, — entirely so outside the consideration of the Earth. But no solid is absolutely rigid, and the action of one body upon another must cause mutual deformation of figure and give rise to tides in the two masses. Darwin has shown[1] that this tidal action is an important cosmic factor, one which has played as constructive a part in the evolution of things as gravitation itself.

Not only were the planets not rigid in the past; they are not rigid to-day. So far as we can judge, all the planets behave as plastic bodies at the present moment. So great are the masses that, even in the case of the denser and cooler ones, deformation of figure seems to be what fluidity and rotary conditions would require. They are, therefore, fit subjects for tidal action.

Tides — how caused.

Owing to the great importance of the subject, and to the fact that the explanation given of it in almost all the text-books is erroneous, I shall present it to you with some pains, the more so that the action may, I think, be outlined quite

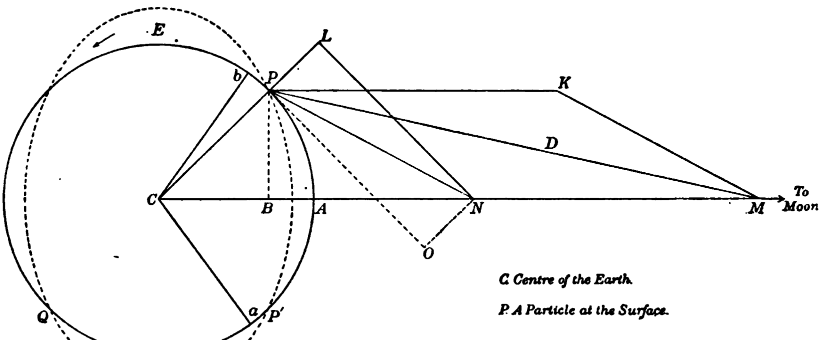
[1] *Pro. Roy. Soc.* 1878–81.

simply. The prestige of Sir Isaac Newton's name is responsible for the inertia which still carries the usual explanation rolling down the ages. He attempted to explain the tides statically, and the account he gave has been blindly copied and perpetuated. But the problem is not a static, but a kinematic one; the body acted on is in motion at the time of the action, and this entirely changes the result. Let me give you an analogous instance of the impossibility of treating a problem of motion as if it were one of rest. The precession of the equinoxes is a case in point, and may be seen in a gyroscope. If a weight be hung on the axis of the wheel while the latter is at rest, the wheel instantly turns into the horizontal plane and stays there. This is a case of statics. If now the wheel be set in motion, however slightly, the wheel, instead of lying down in the plane of the pull once and for all, simply rotates in space without any change of inclination whatever. This is a case of kinematics. Kinematic questions always thus differ from static ones.

Nor can the motion be tacked on afterward, as simultaneity is of the essence of the problem. If the effect of the Earth's rotation was merely to carry forward the crest of the tide through friction, it is the deep-water tides, — those in water over 12¾ miles deep, not the shallow, — that would be nearest under the Moon.

Disturbing force.

Consider a body revolving freely around another in a circle, and disturbed in this motion by a third. This is the case with any particle of the ocean when we neglect pressure and friction. Connect the three bodies by lines, and, keeping their directions, increase their lengths inversely



PERTURBATIVE ACTION
EXEMPLIFYING THE ORIGIN OF THE TIDES.

FIG. VII.

as their squares, and join the ends. The disturbing force will be represented by the connecting line, on the principle of the composition of forces.

$$\frac{CM^2}{PM^2} = \frac{PM}{NM};$$

whence *PN* represents in amount and direction the disturbing or tide-raising force.

If M be far away compared with CP,

$$BN = 2CB = 2p, \text{ say;}$$

for since
$$PM = BM = D$$
and
$$CB = p,$$
$$\frac{(D+p)^2}{D^2} = \frac{D}{NM};$$
whence
$$NM = D - 2p,$$
whence
$$BM - NM = BN = 2p.$$

The tide-raising force PN may be resolved into a normal disturbing force PL and a tangential disturbing force LN. From the fact that BN is always twice CB, we find for the vanishing points of the normal force a and b, those where the angle $BCP = 54° \; 44'$. The whole disturbing force is there tangential.

Now consider the action of the two components; first, that of the tangential factor. At F, the whole force is normal and acting inward. From its minimum here the tangential force rises to a maximum at a, where it comprises the whole force. It then subsides to zero at A. During this quadrant it has been urging the particle onward in its own direction of movement FA. At A, it changes sign and becomes a retarding force, which attains its maximum at b, and then sinks to zero again at E.

In consequence, the velocity of the particle due to the disturbing force is a maximum at *A* — because the force has been adding increments to it up to this point — and a minimum at *F* and *E*. The particle, by traveling fast, lessens the curvature of its path about *C*, since the pull from *C* has less time to act; and reversely by traveling slowly it increases this curvature. In consequence, then, of this component, the path is flattened at *A* and bulged at *E*.

The normal component acts inward at *F* and is proportional to *CP*. It helps the central force at *F*, and curves the path the more. At *a* it vanishes, and is then reversed, acting outward or against the gravity of *C*. It thus lessens the curvature from *a* to *b*. It thus conspires completely with the tangential component; and the two together squeeze the orbit into an ellipse with its longer diameter at right angles to the line joining *C* to *M*.

Tide analogous to moon's variation.

The tidal action on a particle of the ocean is thus precisely the same, neglecting pressure and friction, as that of the Sun upon the Moon's orbit. This deformation of the Moon's orbit was detected, probably by Aboul Wefa, nine centuries ago. It is called the Moon's variation. Thus the tidal wave and the variation are analogous exhibitions of the same force.

Effect of friction.

Friction now comes in to modify the result. At *F*, in consequence of the tide-raising force, the particle is traveling less rapidly than the rest of the Earth. Friction, therefore, urges it on and increases its tangential velocity up to some point *P'*, where its speed becomes equal to the mean speed of the earth. After this, its speed being greater than the Earth's, friction retards it, until it again becomes the mean at *P*. Then friction begins again to accelerate it.

In consequence, the particle is accelerated from *Q* to *P'*, retarded from *P'* to *P*, and then accelerated again. From *A* on, friction thus helps the retarding tangential force, and the Earth causes the particle to turn the corner of the ellipse at *E* sooner than it otherwise would. The tangential force thus reaches its maximum earlier, and the crest of the tide is thus shifted from *E* backward to some point *P*.

On the Earth, in the case of the ocean, we are dealing with superficial tides. In celestial mechanics, it is the substantial tides, or tides of the whole body, with which we are concerned. The latter are immensely the more potent. As the tidal crest lies ahead of the line joining the two bodies, the Sun or the Moon is constantly trying to pull it back into this line, while the Earth is striving by friction to set it at right angles to the line. The bulge, therefore, acts as a brake upon the Earth's rotation, and must continue so to act until the Earth's rotation and revolution coincide.

Tide-generating force.

Now let us determine the tide-generating force[1]:—

Let M = mass of the Earth;
m = mass of the Moon;
x, y, z = the coördinates of the Moon referred to the Earth's centre;
r = its distance;
ξ, η, ζ = the coördinates of the particle referred to the Earth's centre;
ρ = its distance.

Then the Earth describes an ellipse round the centre of inertia of the Earth and Moon, and its acceleration is $\frac{m}{r^2}$ toward this centre.

To bring it to rest, we must apply to it an acceleration, $-\frac{m}{r^2}$, of which the accelerations along the coördinates are,—

$$-\frac{m}{r^2}\cdot\frac{x}{r},\quad -\frac{m}{r^2}\cdot\frac{y}{r},\quad -\frac{m}{r^2}\cdot\frac{z}{r}.$$

Now $$\cos z = \frac{x}{r}\cdot\frac{\xi}{\rho} + \frac{y}{r}\cdot\frac{\eta}{\rho} + \frac{z}{r}\cdot\frac{\zeta}{\rho}$$

and $$r\rho\cos z = x\xi + y\eta + z\zeta;$$

[1] After G. H. Darwin. Article in the *Encyclopædia Britannica* on "Tides."

but $-\frac{m\,x}{r^3}$ is the diff. coefficient of $-\frac{m\,x\,\xi}{r^3}$ with regard to ξ, that is the diff. coefficient of $-\frac{m\,\rho}{r^2}\cos z$.

The potential necessary to bring the Earth to rest is then $-\frac{m\,\rho}{r^2}\cos z$.

The potential of M with regard to the particle is $\frac{M}{\rho}$, while the potential of m upon the particle is $\frac{m}{\sqrt{r^2+\rho^2-2\,r\,\rho\cos z}}$ plus a constant. This constant we determine by the condition that the potential at the planet's centre shall be zero, since we are seeking the motion of the particle relative to this centre, and it becomes $\frac{m}{\sqrt{r^2+\rho^2-2\,r\,\rho\cos z}}-\frac{M}{r}$.

Since r is very large compared with ρ, we may advantageously expand the last in powers of $\frac{\rho}{r}$, which gives : —

$$\frac{m}{r}\left[\frac{\rho}{r}\cos z+\frac{\rho^2}{r^2}\left(\frac{3}{2}\cos^2 z-\frac{1}{2}\right)+\right.$$
$$\left.\frac{\rho^3}{r^3}\left(\frac{5}{2}\cos^3 z-\frac{3}{2}\cos z\right)+\text{etc.}\right].$$

The first term cancels with the potential for bringing the Earth to rest, and we have for the whole potential urging the particle, —

$$\frac{M}{\rho}+\frac{m\,\rho^2}{r^3}\left(\frac{3}{2}\cos^2 z-\frac{1}{2}\right)+$$
$$\frac{m\,\rho^3}{r^4}\left(\frac{5}{2}\cos^3 z-\frac{3}{2}\cos z\right).$$

Of this, the first term is the potential of gravity; the subsequent ones the tide-raising potential.

To get the forces, we must differentiate this expression with regard to the position of the particle.

Tide-raising force for different planets.

In order to compare the tide-raising forces on different bodies, we will assume $z = 0$; whence the tide-raising force at its maximum may be expressed in a rapidly converging series, of which the first two terms are $\frac{2m\rho}{r^3} + \frac{3m\rho^2}{r^4}$.

If the affected body be distant compared with its size, the first term is enough, and we see that then the tide-raising force is directly as the radius of the second body, and inversely as the cube of its distance from the first, while also directly as the latter's mass.

But the work done by a force is the product of the force into the space through which it acts, — as, for instance, the lifting a weight a certain distance, — and in a given time the space is itself proportional to the force, whence the work in that time is as the square of the force.

$$fdt = dv = \frac{ds}{dt},$$

whence $$\tfrac{1}{2}ft^2 = s.$$

Whence if the time remain constant the force must vary as the space. For the proportionate work done in a given time by tide-raising forces, we have, then, $\left(\frac{2m\rho}{r^3}+\frac{3m\rho^2}{r^4}\right)^2$, or for most cases sufficiently well, taking only the first term, $\frac{4m^2\rho^2}{r^6}$. That is, it is as the square of the attracting mass and the square of the radius of the affected body directly and inversely as the sixth power of the latter's distance.

WORK DONE BY TIDE-RAISING FORCE IN UNITY OF TIME IN RATIO TO SUN'S ACTION ON THE EARTH TAKEN AS UNITY.

By Sun on:—	$\frac{4m^2\rho^2}{r^6}$ (*approx.*)
Mercury	43.26
Venus	6.60
Earth	1.00
Mars	0.023
Jupiter	0.006
On Earth by:—	$\left[\frac{2m\rho}{r^3}+\frac{3m\rho^2}{r^4}\right]^2$
Sun	1.00
Moon	4.97
On Satellites by their Primaries:—	$\left[\frac{2m\rho}{r^3}+\frac{3m\rho^2}{r^4}\right]^2$
Iapetus	27.6
Callisto	32,549.0
Ganymede	1,385,600.0
Moon	2,374.4

Professor G. H. Darwin has calculated the relative effect of tidal retardation by the Sun on each of the several planets, that upon the Earth being taken as unity, with the accompanying result: —

Planet.	*Number to which Tidal Retardation is Proportional.*
Mercury	1000. (?)
Venus	11. (?)
Earth	1.
Mars	.89
Jupiter	.00005
Saturn	.000020 to .000066

Supposing, then, all the bodies to have started in the race for rotary retardation at the same time, the isochronism of rotation and revolution of Mercury is what was to have been expected. For the previously known facts were: first, that the Moon showed this state of things. Now the relative tide-raising effect in a given time of the Earth on the Moon is 2374; that of the Sun on the Earth being unity. Second, that Iapetus did the same; for this satellite is always much brighter on the western side of Saturn than on the eastern. Such a periodic change of brilliancy would be accounted for by isochronism of rotation and revolution. Now the relative tide-raising effect of Saturn on this satellite is 28.

On the other hand, the Earth's rotation and revolution do not coincide; and the relative effects of Sun and Moon on it are:—

Sun	1.00
Moon	4.97
	5.97

Assuming, therefore, that the retardation began synchronously for all, Mercury, upon whom the effect was 43, should have reached the isochronous condition.

We may note incidentally that Venus on this assumption falls in the debatable ground, since the effect on it is 6.60.

But we do not know either the time of the birth of the Moon nor the relative age of the Earth and Venus. It is quite possible, for aught we know, that Venus may have been subjected to the process practically much longer than the Earth.

It is certainly significant that isochronism ceases just where a first approximation would put it.

Since the date of the detection of Mercury's isochronism by Schiaparelli, the third and fourth satellites of Jupiter, Ganymede and Callisto, have been added to the isochronous list by Mr. Douglass at Flagstaff. These, then, agree with theory. We may safely predict that all the other satellites of Jupiter and Saturn will be found to behave similarly.

Consummation of tidal effect marks the last stage in the planetary career. So soon as identity of rotation and revolution is effected, the planet is placed in a changeless, or largely changeless, state, which, so far as we can conceive, means as a world its death. It now turns the same face, except for libration, in perpetuity to the Sun. Day and night, summer and winter, have ceased to exist. One half of it is forever being baked, the other half forever frozen; and from this condition there is no escape. The planet must remain so until the Sun itself goes out.

Mercury, therefore, represents planetary decrepitude; and the symptoms of this old age are: loss of air; isochronous rotation and revolution; rotundity.

III

MARS

MERCURY presents us one phase of planetary development; Mars another, quite different. The two represent stages in world-life as distinct as those of gray hair and brown in human life.

Mercury old; Mars in middle age.

Whatever the absolute ages of the several planets, their relative ages, as measured intrinsically, decrease pretty steadily with their distance from the Sun. Mercury is old; Mars, middle aged; Jupiter young.

World-life has its earmarks of time as human life has, and betrays them quite as patently.

Lack of atmosphere, colorlessness, changeless attitude toward the Sun, are the signs of old age in a planet. Mercury shows all these tokens of senility. Mars presents a very different picture.

Color a conclusive criterion.

Color is a telltale trait; for it is a sign that surface development still goes on. Lack of atmosphere alone prevents vegetation, and this, coupled with unalterableness of face presented to the Sun, weathers the surface to a neutral gray. Such a body shows but the bleached bones of a once living world.

Now color is conspicuously wanting on Mercury. The disk of the planet is a chiaroscuro of black and white, tones devoid of tints.

Mars a life-supporting world.

Mars is an opal. Colors comparable only to that stone variegate its disk. At top and bottom, collars of pearl-white contrast vividly with light areas of rose-saffron and darker ones of robin's-egg blue. Daylight reveals these colors much better than night, because the contrast of the blue-black sky clothes the disk with yellow it does not really possess, diluting the true tints.

Mars has polar regions, temperate zones, and tropics.

The markings enable the rotation of the planet to be found. The markings move under the observer's eye and yet keep their relative configurations the same, day after day and year after year. They thus reveal the fact that the planet rotates, and by the course of their motion disclose the axis about which the rotation takes place. From the observed data, spherical trigonometry enables us to fix this axis in space and determine its tilt to the plane of the planet's orbit. We thus find that it is inclined to the Martian ecliptic by an angle of 25°, and that the solar day there is 24 hours and 40 minutes long. Thus Mars has both days and seasons, and both days and seasons are practically counterparts of our own. The days are a little longer and the seasons nearly twice as long, reckoned either by Earthly or by Martian days. The orderly succession of day and night, spring, summer, autumn, and winter, are the same there as here.

Seasons accentuated much like ours, and day of about same length.

Now this is no accident. It is a direct consequence of the planet's size and of its position in the solar family. That, however, the circumstances of the Earth and Mars should chance to agree so nearly in quantity as well as quality, we as yet lack the data to explain.

Size, or rather lack of it, has done something else for Mars. It has reduced the atmospheric blanket that covers the planet's body. It did this both at the start and subsequently. If the planets set out with atmospheres in proportion to their masses, a small planet having a greater surface in proportion to its mass would not have this surface so thickly covered, and its lesser gravity would further spread this out skywards.

Scant atmosphere.

Surface being as $4\pi r^2$, while mass is as $\frac{4}{3}\pi r^3$, the one is to the other, surface to mass, as

$$\frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}.$$

The ratio of surface to mass increases, therefore, inversely as the radius of the body.

In the next place, its gravity could control only a much smaller velocity at its surface, thus making the critical velocity beyond which a particle would pass off into space much less. By the kinetic theory of gases, a certain number of particles will in a given time attain the critical velocity, and the more the lower the critical velocity. Thus, from the planet that hath not shall be taken away even that which it hath.

In consequence, on Mars the density of the air at the surface of the planet at the start was probably not denser than one-seventh of our own, or more rare than that at the top of our loftiest mountains, and now probably is rarer even than this, owing to the greater speed with which it has been lost.

Rate of loss differs with different gases.

The rate at which the different gases would be lost differs. The curve of probability shows that they would disappear much more rapidly than the ratio of their speeds. Water vapor would go long before atmospheric air.

MAXWELL'S LAW.

The possible values which the components of the molecular velocities can assume are distributed among the molecules in question, according to the same law by which the possible errors of observation are by the method of least squares distributed among the observations.

The number of molecules traveling at speed u is given by the equation, —

$$N\sqrt{\frac{km}{\pi}}e^{-kmu^2}du = dy,$$

just as the probability of an error is given by the equation, —

$$y = \frac{h}{\sqrt{\pi}}e^{-h^2x^2}.$$

VALUES OF THE SPEEDS.

G = mean value of speed in metres per second.
G' = mean value of speed in miles per second.

	G	G'
Hydrogen	1838	1.14
Water vapor	614	0.38
Nitrogen	492	0.31
Atmospheric air	485	0.30
Oxygen	461	0.29
Carbon dioxide	392	0.24
Cyanogen	361	0.22

These speeds are got from the consideration that the energy, from which follows the temperature, is the same in the two gases; and, therefore, that

$$\frac{1}{2}mv^2 = \frac{1}{2}m_1v_1^2;$$

and, therefore, the speed of the molecule is inversely as the square root of the atomic weight.

So far theory. Now it is not a little interesting that observation supports this. That air still exists on Mars, oxygen, nitrogen, and carbonic acid, Air on Mars.

is certain because of the changes which we can see going on in the surface markings; for without air no change could take place, and changes are indisputable. Water is relatively scarce.

Change in polar caps.

That change goes on upon the planet's surface has been known for a long time. The polar caps were the first telltale. Sir William Herschel, at the end of the eighteenth century, observed that they waxed and waned periodically, and that their period was timed to that of the planet's year. They were therefore seasonal phenomena.

They behaved like ice and snow, and this they are generally supposed to be. Some astronomers find difficulty in conceiving of enough heat on Mars to permit them to be water, and carbonic acid has been suggested instead. But certain phenomena connected with the melting prove that carbonic acid cannot be the substance. The evidence is now very strong that they are what they look to be, and that the necessary heat will somehow be explained.

Pre-Schiaparellian knowledge and ideas.

Up to the time of Schiaparelli, not much beyond this behavior of the polar caps and the general permanency of the dark and light markings was known about the planet. Its physical condition was likened to the Earth's, the white patches being polar snows, the dark markings oceans and seas, and the light markings land.

In fundamentals, indeed, Mars shows a general similarity to the Earth; but in subsequent characteristics it betrays a most interesting dissimilarity. It is the dissimilarity that modern study has specially brought out.

Mars intrinsically older than the Earth.

The cause of the dissimilarity springs from the planet's size. The less mass of Mars did not permit it initially to present so fertile a field for development. Mere size entirely alters physical possibilities. In the next place, its dwarfing caused it to age quicker than the Earth.

Schiaparelli's discovery.

Our knowledge of the planets, and especially of Mars, has advanced greatly within the last quarter of a century. The first steps of this advance we owe, not to instruments, but to the genius of one man, the Italian astronomer Schiaparelli. In 1877 he began to observe Mars, and at once showed a keenness of vision surpassing that of any previous observer and a susceptibility to impressions surpassing even his acuteness of sight. It was not so much a matter of eye as of brain. For it turns out now, after the fact, that several of his phenomena had been dimly seen and recorded before, but without that understanding which made of them stepping-stones to further results.

His object was to map the planet micrometrically. But in the course of his mapping he became aware of some curious markings: dark bands seaming the surface of the light areas, or so-called continents. These he named *canali*, or channels; for he, in company with every one else, at the time believed the dark regions to be seas.

Having got the hint, for it was scarcely more than that, during his first season, the opposition of 1877, he then showed that element of genius without which very little is ever accomplished, the persistence to follow up a clue. As Mars came round again he attacked the planet in the light of what he had already learnt, and first confirmed and then extended his discovery. This he continued to do at each succeeding opposition. The more he studied, the stranger grew the phenomena he detected. And it is to his everlasting credit that he did this in the face of the skepticism and denial of practically the whole astronomic world. He won. The voices that ridiculed him are all silent now. To-day the canals of Mars are well-recognized astronomic facts, and constitute one of the most epoch-making astronomic discoveries of the nineteenth century.

Through a complete cycle of oppositions, that is, from the nearest to the most remote and round to the nearest again, a period of fifteen years, Schiaparelli continued to study these curious phenomena, having them practically all to himself. Indeed, his grand isolation in the quest makes one of the finest and saddest chapters in the history of discovery. In the course of these solitary years he came to see the canals better, and they grew, on improving acquaintance, steadily more strange. He found that they were far more regular than he had at first thought, and he noted that they were dependent in appearance upon the season of the planet's year. So, likewise, were the large dark markings, and he attributed the behavior of both to a seasonal shift of water over the surface.

His theory of the planet's physical condition, derived from his observations, was as follows: that the polar caps were ice and snow; that the blue-green areas were seas and the reddish-ochre ones land; that the canals were natural water-channels or straits honeycombing the land and cutting it up into a patchwork of large islands, a sort of natural Venice on a world-wide scale; and finally that the surface was subject to annual or semiannual inundations and dryings-up, timed to the melting of the polar caps.

Schiaparelli retired practically in 1892, though not formally till a little later. His work was taken up by other hands, and the impetus he gave the matter has resulted in a knowledge of Mars which has quite revolutionized even the conception he bequeathed of the planet.

Methods of observation.

Before proceeding to post-Schiaparellian work, it may interest you to know how the phenomena in question have been detected, and what they look like when seen.

Contrary to what the layman thinks, the size of the instrument is the least important factor in the process. As in most things, the man is the essential machine; and next in desirability to the presence of man is the absence of atmosphere. In good air, with fair attention, the canals are not very difficult objects. Indeed, the surprise is that they were not detected long ago. Under suitable atmospheric conditions a four-inch glass will show them perfectly. Steady air is one essential; steady study another.

The canals.

In appearance they are unlike any other phenomena presented in the heavens. Pale pencil lines, deepening on occasion to India ink, seem to cobweb the continents. Their tone depends on the seeing, in the first place, and on the season, in the second. Their width is invariable throughout, and their directness something striking. Measurable width they have not; it is only by

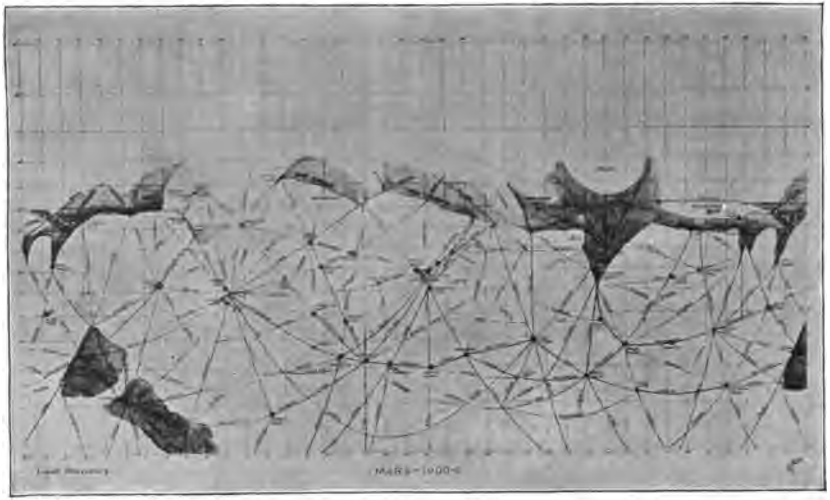


FIG. VIII. MAP OF MARS.

depth of tint that their importance is inferred. But their most amazing attribute is their geometric character. They seem to be generally arcs of great circles drawn from certain salient points on the planet's surface to certain other equally salient ones.

Their number appears to be legion. Schiaparelli discovered 104. But the better the planet is seen the more of them come out. About 350 have now been mapped at Flagstaff, and the number is only limited by our penetration. Like the asteroids, the larger ones have already been detected. Each opposition now brings out smaller and smaller specimens.

Their seasonal character.

But now comes a most interesting fact connected with them which was discovered by Schiaparelli and found equally true at Flagstaff. They are not always equally visible. Sometimes they are conspicuous, sometimes scarcely discernible even to a practiced eye. And this is not mere matter of distance. The best time for seeing the planet is not the best time for detecting the canals.

At certain oppositions we pass the planet at close quarters, at certain others a good way off. The close approaches are called favorable oppositions, the distant encounters unfavorable ones.

But the latter are not so unfavorable as they are thought. For another factor beside nearness affects the reckoning. The planet's axis is tilted to the plane of its orbit at an angle of 25°, and is so faced that the southern hemisphere is presented to us at the time of closest approach. Now the canals lie chiefly in the northern hemisphere. In the next place, it is then the northern winter, and careful comparison reveals the fact that the conspicuousness of a canal is a function of the Martian time of year, becoming pronounced in summer and fading out in winter.

This is one reason why the canals so long eluded astronomers. They were not looked for at the proper time.

"Seas" not seas.

The first important post-Schiaparellian advance was made in the dark regions of the planet.

For two centuries the dark regions were held to be seas. It became evident, however, from Pickering's observations in 1892 that the great part of them could not be such. In 1894, at Flagstaff, it further became evident that no part of them could be water. From the way in which the clarification of the dark regions progressed with the planet's seasons, it had become patent that the bodily transference of substance, such, for instance, as water, from one place to another, could not account for the phenomena. For the decrease in one locality was not offset by the increase in others. As the quantity of the change, positive and negative, did not balance, the change could not be due to a shift of matter. It must, therefore, be ascribable to a transformation of matter. And the only thing of suitable conduct and proper local color to show the phenomena was vegetation. The "seas" were not seas, but probably areas of vegetation.

Oases. The next significant discovery was the detection of the oases, or small round black spots that dot the planet's surface. These were initially seen as such by W. H. Pickering, at Arequipa, in 1892. Pickering called them lakes, but for a reason which will appear later it seems more proper to consider them oases. Quite as singular a feature as the canals, they prove to be as universal a one. They are the more difficult of detection; which is the reason they were recognized later. Schiaparelli told the writer that he had himself suspected them, but could not make sure.

Just as the canals form a mesh over the disk, so the oases make the knots where the lines of the network cross. To them, in short, the canals rendezvous. The number of lines which thus come together at one and the same point is sometimes considerable. Nine meet at the Phœnix lake, eleven at the Trivium Charontis, and no less than seventeen at the Ascræus Lacus at the top of Ceraunius. Nor, so far as can be seen, is any important junction without its spot. Their bearing upon the explanation of the canals is at once evident.

In character the oases are, when well seen, very small and very dark. Too small to disclose distinctive color, they are the most deeply complexioned detail upon the disk, and presumably blue. It is only in poor air that they show large and diffuse. About three degrees in diameter and seemingly quite round as a rule, they must be 100 miles across, and, for all their minuteness, cover a goodly area of ground.

They seem to share the same seasonal transformation with all the other markings.

Canals and oases in the dark regions.

The next step was the discovery of canals in the dark regions of the planet. Streaks in these regions were seen in 1892 at Arequipa and at the Lick Observatory, much as Dawes had seen streaks in the light ones thirty years before. But in 1894, at Flagstaff, Mr. Douglass found that the streaks were not irregular markings, but a system of lines possessing the same singular characteristics which distinguish and differentiate the "canals" in the light regions from other celestial phenomena. In short, he detected in the dark regions what Schiaparelli had detected in the light. Counterparting exactly the network over the light areas, a mesh of similar lines overspread the blue-green areas. The lines were of uniform width, of unswerving directness, and went from definite points to other equally determinate ones. These points were always of geographic importance. They were at the ends of "seas," at the bottom of "bays," or at points on the "coast-line" where canals debouched. The lines connected these topographical centres, crossing one another in the process, and at the junctions there showed, just as in the light areas, dark round spots.

Instantly to be deduced from such engraving was that the "seas" were not bodies of water. We knew this already, as I have shown; but the evidence was valuable in completely convincing those who require more than mediate proof. Permanent lines cannot be writ on water. The seas lost their character forever.

The absence of any bodies of water outside of the temporary polar sea introduces a far-reaching difference between Mars and the Earth. On Earth three quarters of the surface is water; on

Mars all is land. Instead of having more sea than it can use, the planet must be in straits for the article. Its whole supply comes from the annual melting of the polar caps.

The canal system of the dark regions not only

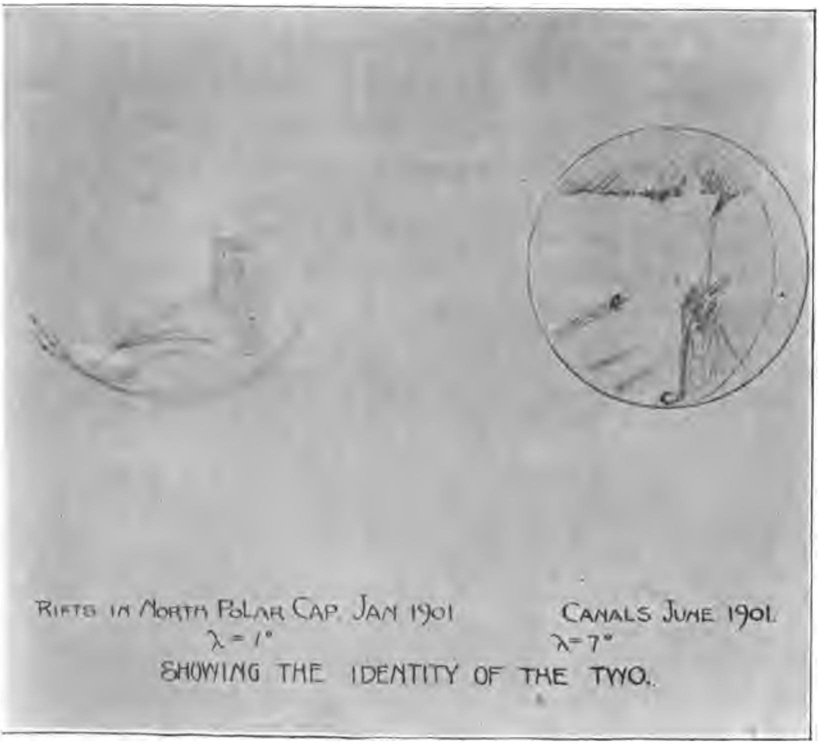


FIG. IX.

Two systems. resembles the system in the light; the one joins on to the other. The points where the system in the light areas strike the dark are the points from which the canals in the dark regions set out. The two are thus but parts of one world-wide whole. Whatever purpose the one subserves is thus taken up and extended by the other.

Nor does the communication come to an end in the dark regions. From the southern portions of these, in the southern hemisphere, other canals run straight into the polar cap; in the northern hemisphere, similarly, canals penetrate to the most northern limit of the snow.

Lastly, the rifts which appear in the caps during the process of melting turn out to be where subsequently are seen canals. Now, as there are no mountains on Mars, differences of level cannot be a cause of melting; areas of vegetation could.

Summation. We may sum up our present knowledge of the surface conditions of the planet as follows: —

(1) Change takes place upon the planet's surface; this proves the presence there of an atmosphere.

(2) The limb-light, the apparent evidence of a twilight, and the albedo, all point to a density for this atmosphere very much less than our own.

(3) The polar caps melt in their summer and accumulate in their winter, thus showing themselves to be seasonal in character.

(4) As they melt, they are bordered by a blue belt, which retreats with them. This negatives carbonic acid as the substance composing them, and leaves to our knowledge only water as a possible explanation.

(5) Their extensive melting shows their quantity to be inconsiderable, and points to a dearth of water.

(6) Comparison with previous observations shows the melting to occur in the same consecutive places year after year. The melting is thus a thing which can be locally counted on.

(7) The greatest local melting is just south of the largest dark (blue-green) regions, the bays in the polar sea in these longitudes being the largest.

(8) The dark regions are subject to a wave of seasonal changes ;

(9) which follows upon the melting of the cap. They darken in early summer and fade out in their autumn.

(10) The dark regions are not seas : first, because in Professor W. H. Pickering's experiments their light showed no trace of polarization, while that of the polar sea did ;

(11) second, because the quantity of the darkening is not offset by the synchronous lightening elsewhere. It cannot therefore be due to shift of substance;

(12) third, because they are seamed by a canal system counterparting that of the light areas, permanent in place.

(13) Extension of this shows that there are no permanent bodies of water on the planet.

(14) All the phenomena are accounted for by supposing them to be areas of vegetation.

(15) The polar sea being a temporary affair, the water from it is fresh.

(16) Observations on the terminator reveal no mountains on Mars, the details of the observations being incompatible with such supposition;

(17) but do reveal apparently clouds, which, however, are rare, and are chiefly visible at sunrise and sunset,

(18) and seem connected with the heat equator.

(19) The bright areas look and behave like deserts.

(20) In their winter, the south temperate light regions are covered by a white veil, which may be hoar-frost or may be cloud.

(21) Very brilliant patches appear also in the equatorial light regions that last for weeks, and seem independent of diurnal conditions.

(22) They appear always in the same places.

(23) A spring haze surrounds the polar caps during certain months, outside of and distinct from the cap itself.

(24) A progressive change of darkening sweeps over the planet's face from pole to pole semi-annually, beginning with the cap, and developing as vegetation would down the disk.

Conclusions as to physical condition.

These phenomena lead to the conclusion that the polar caps are masses of snow and ice; that the light areas are deserts; that the blue-green areas are tracts of vegetation; that there are no permanent bodies of water on the planet, and very little water in any form; that the surface is remarkably flat; that the temperature is moderately high by day but low at night; that it is fairly warm in summer but cold in winter; and that the seasonal change of the vegetation is marked even at our distance away.

To these conclusions we are led by the general aspect and behavior of the planet's disk. We have reached them without reference to the canals considered in themselves, and we should continue to put faith in them were the canals, with all their strange characteristics, blotted from existence. Unbeholden, then, to the canals for this conclusion, we are the more impressed to find that the supposition that the "canals" are not the result of chance falls completely in line with our result.

Water is very scarce on the planet, and is absolutely essential to life. Vegetation exists there, and it is therefore highly probable that organic life is to be found there, too. This becomes *a posteriori* probable, when we behold a system of lines inexplicable on any other ground and precisely what would be needed for the diffusion of water over the planet's surface.

What we find is this: —

(25) A network of fine dark lines meshing the deserts.

(26) The lines are uniform throughout and from five to thirty-five miles in width,[1]

(27) and hundreds, sometimes thousands of miles long,

(28) usually, if not always, following arcs of great circles,

(29) starting from topographically important points in the dark regions,

(30) and traveling to other equally conspicuous points;

(31) both terminals show dark spots, a caret in the coastline and what seems a round spot in the desert;

[1] Tests by the writer on telegraph lines show that a line can be seen, *owing to its length*, when its width is 2″.5, to the naked eye. This would mean about 5 miles on Mars.

(32) all the way from three to seventeen "canals" will converge upon the same spot;

(33) the spots are perhaps a hundred miles in diameter, and their number is very great;

(34) the dark regions are meshed by a similar network;

(35) the points of departure of both are the same;

(36) similar centring spots show in the dark areas, darker than their background;

(37) with the dark network "canals" others connect, running to the edge of the extreme melting limits of both caps;

(38) the lines are seasonal phenomena, developing after the melting of their respective polar cap and fading out later;

(39) those in the polar regions occupy the place of earlier rifts in the snow-field, as if the ground were there thawed by vegetation.

They are of uniform width; that is, they waste nothing in breadth. Whatever breadth is necessary is used, and no more, and that is retained throughout. They go directly from certain conspicuously probable points to certain others. If we were obliged to connect the planet by a system of intercommunication, it is precisely those points we should ourselves select.

In addition to the departure points on the borders of the dark regions which are provided by nature are a host of others not apparently so originated. These are the round black dots,—the oases. They are found at the intersections of the lines. How important they are in the planet's economy is to be inferred from the host of canals each of them receives. Four, very rarely three, is the minimum number of approaches or departures from them, and this number rises in the case of Ceraunius to seventeen. Even London hardly has this number of railway lines entering and leaving it. It is not too much to suppose, though as yet we cannot count it more than a conjecture, that the oases serve some such purpose as our cities and are centres of population.

From this, we add to our list of conclusions,—that the canals are artificial, and therefore imply organic intelligent life upon the planet.

Our synthesis leads, then, to the conclusion that Mars is circumstanced like ourselves in the midway of planetary existence, but that the planet has advanced further on the road to old age and death than we have yet done.

That its world-life was, in any but the broadest sense, an analogue of our own, is certainly not the case. Its career began under different physical conditions, owing to its size, ran more rapidly through its successive stages, again owing to its size, and will come to an end sooner for the same reason.

As a detail of this, life on Mars must take on a very different guise from what it wears on Earth. It is certain that there can be no men there; that is as certain as anything well can be. But this does not preclude a local intelligence equal to, and perhaps easily superior to, our own. We seem to have evidence that something of the sort does exist there at the present moment, and has made imprint there of its existence far exceeding anything we have yet left upon mother Earth.

In conclusion, let me warn you to beware of two opposite errors; of letting your imagination soar unballasted by fact, and, on the other hand, of shackling it so stolidly that it loses all incentive to rise. You may come to grief through the first process; you will never get anywhere by the second. Take general mechanical principles for compass and then follow your observations. Imagination is as vital to any advance in science as learning and precision are essential for starting points.

IV

SATURN AND ITS SYSTEM

Saturn.

SATURN marked to the ancients the outer boundary of the solar system. From its slow motion, they rightly conjectured it to be the farthest away of all the "wanderers," and wrongly to be sinister in intent. Our word "saturnine" expresses the feeling it inspired.

In the telescope, Saturn is undoubtedly the most immediately impressive object in the heavens. Few persons can be shown the planet for the first time without an exclamation. To see it sail into the field of view, its great ball diademed by an elliptic ring, and carrying with it a retinue of star-points set against the blue-black background of the sky, gives the most prosaic a sensation.

Saturn's self we shall leave till we come to speak of Jupiter (in the next chapter); and shall here consider the two systems of bodies dependent on it, — its rings and its satellites.

The ring system.

Unique, so far as we know, is that appanage of Saturn which makes the planet so superb a sight, — the ring system. It baffled Galileo with his opera-glass, who first saw the planet triform, and then, to his surprise, marked the two smaller bodies disappear, as if Saturn had indeed eaten his offspring.

Crowning the planet's equator are several concentric flat rings of light. Three are usually distinguished, known as *A*, the outer ring; *B*, the middle ring; and *C*, the inner or dusky ring. The outer, *A*, has an extreme radius of about 85,700 miles. It is 12,000 miles across, and is separated from *B* by a dark space 3400 miles wide, known as Cassini's division. *B*, the broadest and brightest of the rings, is 17,000 miles in width, and is joined without perceptible interval by *C*, which is much fainter, resembling a crêpe veil stretched from the inner edge of *B*, 9500 miles toward the planet, from whose limb it is sundered by a gap of between 7000 and 8000 miles.

Two thirds way from the outer to the inner edge of *A* is another division or dark line, much narrower than Cassini's, and sometimes nearly invisible, known as Encke's division, though suspected before by Short.

Constitution of the rings.

Edward Roche, in 1848, was the first to show that the rings were composed of discrete particles, — mere dust and ashes. He drew this consequence directly from his investigations on the minimum distance a small fluid satellite may safely approach a fluid primary; for within a certain distance the differential or tidal pull of the planet must disrupt the satellite. This distance is called Roche's limit.

For equal densities of planet and satellite, Roche's limit is 2.44 times the planet's radius; for unequal densities, as $\sqrt[3]{\frac{d}{d'}} \times 2.44$, where d is the density of the primary; d' of the satellite.

Saturn's system offers the only instance where matter circulates within the limit, and Roche stated distinctly that the rings, therefore, must be mere meteoric stones.

Even Laplace had shown that the rings must be broken up for stability's sake into several narrow ones, each revolving at its own rate. Pierce proved that they could in no case be solid. Maxwell then demonstrated that they could not be so much as liquid, as disrupting waves would be set up, but must consist of a swarm of small bodies, — brickbats he likened them to, — each pursuing its own path. What the spectroscope in Keeler's ingenious hands made visible to the eye had thus been known to mechanics from the time of Laplace.

These flights of small bodies are so exactly in one plane that they vanish when the rings are turned edgewise to the Earth. Their lustre shows them to be relatively densely packed, so that collisions among them must be not infrequent. In consequence of this, Maxwell predicted that they would eventually be forced both out or in, and in part fall upon the ball, in part be driven farther from the planet. Certainly such must ultimately happen; but the evidence is not conclusive that either process has yet been observed.

All in the same plane.

The spectroscope shows that, unlike Saturn, they carry no air with them. This, from their minute size, was to be expected on the kinetic theory of gases and the clever deduction from it as to the atmosphere a body may retain, made by Johnstone Stoney.

No air about them.

To attempt to account for their dimensions and divisions might at first seem hopeless. Why *A* is made up of an outer and an inner portion parted by Encke's streak; why *B* is sundered from *A* by Cassini's division; and why *C* is sharply contrasted with *B* at its inner edge, sound like difficult questions. But nothing in celestial mechanics is the outcome of chance, and this is no exception to the rule.

Gaps in them.

To begin with, Roche's limit falls just at the

Roche's limit. outer edge of the system, supposing the density of the satellite to be $\frac{8}{9}$ of the primary's. Now the satellites of Saturn are certainly a little denser than the planet. From our present values of its mass and volume, Titan's density comes out .24. This, then, is what has limited the system externally.

For the rest of it, another force has proved fashioner.

Commensurate periods. Our mathematics do not permit us to solve rigorously the problem of three bodies; that is, the motion of a first revolving round a second and perturbed by a third. We have to have recourse to approximations in series. We can thus determine to any degree of accuracy the result. Now the perturbative effect produced by a third body upon the major axis of a second revolving in its own plane may be expressed by a series developed in terms depending on powers of the eccentricity and cosines of multiple arcs of the mean motions. The typical form of one of these terms is

$$P \frac{ca^2 n}{pn - qn'} e^l e^m \cos \overline{pn - qn' - Q},$$

where P is a function of a and a', the radii vectores.

From this, it appears that if p and q are nearly in the inverse ratio of the mean motions,

$$pn - qn' \text{ is nearly } 0,$$

and the term has a large coefficient, and therefore a large value.

If, then, the mean motions, and therefore the periods, of perturber and perturbed are commensurable, the disturbing effect upon the major axes of each will be great. The major axes will be altered until the periods cease to be commensurable, and it will be long before perturbation brings them back to commensurability again.

Greatest effect with the smallest ratio.

Furthermore, the least value $l + m$ can have is $p - q$, while the period of the action of the term is $\frac{2\pi}{pn - qn'}$; whence the greatest term is when p and q are both as small as possible, since conjunctions will occur oftener in proportion as q is small.

Effect geometrically considered.

Geometrically, the effect can be seen in the following way. Clearly, the disturbing pull is greatest when the two bodies are in conjunction, and so long as the periods are incommensurable, conjunctions will occur in different parts of the orbit successively, and thus neutralize one another's effect upon the major axes. But if the periods of the two bodies be commensurable, conjunctions will occur in the same place over and over again, and the major axes will be altered there without compensatory alterations elsewhere; and this will go on until the major axes are so altered that commensurability of period, which depends on the major axis, ceases. Then the bodies will cease to affect each other forcibly. They will gradually meet each other elsewhere, finally oppositely to what they did at first, and the action first produced will be as gradually undone; but it will be very long before the major axes attain their original value again; then they will pass rapidly through them once more in the reverse way.

If, then, the periods of the two bodies are commensurable, they will not appear to be so, since their major axes will stay commensurate but a brief time compared with the time they are out.

Gaps due to change of major axis.

Now, if we have a swarm of bodies revolving at various distances round a central mass, and disturbed by a third, the third will seem, in consequence of this, to sweep out spaces where otherwise bodies would revolve in times commensurate with its own. Jupiter has done this very thing in the case of the asteroids, striping the zone with vacant belts. Calculation alone reveals this, as the asteroids are too few to disclose the fact to the eye. But in the rings of Saturn we can actually see the empty places. The gaps in the rings are shown in the following table and in the accompanying picture of the ring system: —

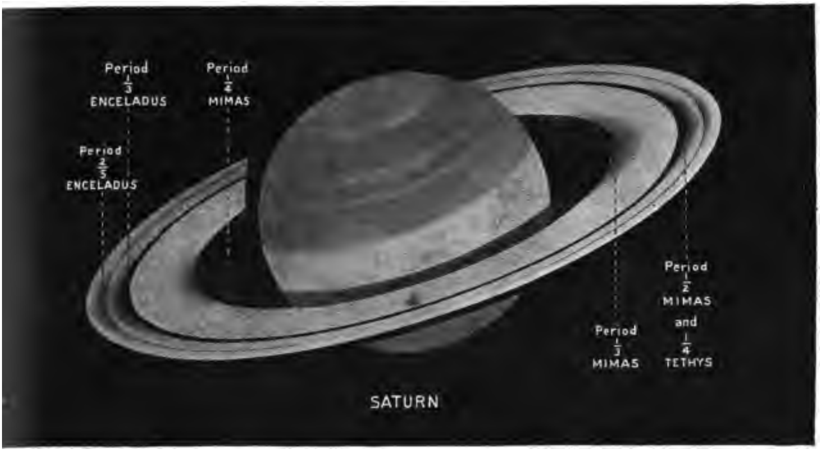


FIG. X. SATURN'S RINGS.

	Old Determination.		*New Determination.*	
Outer radius outer ring *A*	84000		85700	
Encke's division . .		80000		81010
Inner radius outer ring *A*	74000		75300	
Cassini's division . .		73200		73620
Outer radius ring *B* . .	72400		71900	
Inner radius ring *B* . .	56200		54700	
Outer radius ring *C* . .	56200		54700	
Inner radius ring *C* . .	46700		43900	
Planet radius	37500		37000	

Gaps occasioned by Mimas.

Let us note these gaps and edges and then calculate what perturbing effect the satellites would exert. The satellites which would have the greatest perturbative action on the rings is Mimas, his effect being more than three times that of Enceladus and more than twice that of Tethys.

The equations of motion are for x —

$$\frac{d^2x}{dt^2} = -\frac{x}{r^3} + \frac{m'(x'-x)}{(r'-r)^3} - \frac{m'x'}{r'^3},$$

of which the first is the direct force of the central body, whose mass is taken as 1, upon m; and the other terms are the perturbing force of m' on m.

Assuming the three to be in conjunction, this last becomes $m'\left(\frac{1}{\rho^3} - \frac{1}{r'^3}\right)$, where $\rho = r' - r$. Supposing m to be 74,000 miles from the centre of Saturn, and Mimas, Enceladus, and Tethys at 117,000, 150,000, and 186,000 miles respectively, and taking the masses as proportionate to their volumes, their radii being taken as 400, 400, and 600 miles, we find for their relative perturbative effects: —

Mimas	299
Enceladus	82
Tethys	110

The action of the others is smaller still. Now the major axis of a part of the ring which has a period commensurate with that of Mimas may be found from the formula —

$$\frac{T^2}{T_{\prime}^2} = \frac{a^3}{a_{\prime}^3},$$

Kepler's third law. Beginning, then, with the simplest, and therefore the most potent ratio, $\frac{1}{2}$, we find 73,600 miles for the major axis of a particle whose period is $\frac{1}{2}$ that of Mimas. This distance falls almost exactly in the centre of Cassini's division.

Proceeding to the next simplest ratio, $\frac{1}{3}$ of Mimas's period, the corresponding distance comes out 56,170 miles. This is the distance from the centre of the planet to the inner edge of ring *B*.

Again, $\frac{1}{4}$ of the period of Mimas gives 46,370 miles. This is not far from the radius of the inner edge of the dark ring. So much for the action of Mimas.

By Enceladus.

The major axis of one half the period of Enceladus falls without the system, but the major axis of one third the period occurs at 72,090 miles. This is not far from the inner edge of Cassini's division. But the striking coincidence with Enceladus is that the distance corresponding to $\frac{3}{5}$ of his period lies at 81,400 miles, or at Encke's division.

To Tethys.

For Tethys, the only commensurable ratio is $\frac{1}{4}$. This makes the distance fall at Cassini's division.

Thus Mimas, aided by Tethys, has been the divider of the rings into *A*, *B*, and *C;* while Enceladus has subdivided *A*.

Satellites.

Not less interesting mechanically is Saturn's satellite system. Eight of these bodies are positively known, distanced from Saturn and diametered as follows: —

RELATIVE SIZE AND POSITION OF THE SATELLITES.

No.	*Name.*	*Diameter in Miles.*	*Distance from Saturn in Miles.*
I.	Mimas	800	117,000
II.	Enceladus	800	150,000
III.	Tethys	1,200	186,000
IV.	Dione	1,100	238,000
V.	Rhea	1,500	332,000
VI.	Titan	3,500	771,000
VII.	Hyperion	500	934,000
VIII.	Iapetus	2,000	2,225,000

Relative position of the masses of the satellites.

It will be seen that the largest — Titan — occupies a central position in the line. This might seem accidental until one recalls the fact that Jupiter, the largest of the planets, holds the same relative place in the solar system: for the planetary system tabulated in the same way is as follows: —

SOLAR SYSTEM.

No.	*Name.*	*Diameter in Miles.*	*Distance from Sun in Millions of Miles.*
I.	Mercury	3,300	36
II.	Venus	7,630	67
III.	Earth	7,918	93
IV.	Mars	4,220	141
V.	Asteroids	10–500	250
VI.	Jupiter	86,500	483
VII.	Saturn	72,500	886
VIII.	Uranus	31,900	1,782
IX.	Neptune	34,800	2,792

With the hint given by this, at least singular, coincidence, let us examine the other satellite systems. Two others available for comparison present themselves, — that of Jupiter and that of Uranus. Jupiter's system is this: —

No.	*Name.*	*Diameter in Miles.*	*Distance from Jupiter in Miles.*
V.	(Nameless)	100	112,500
I.	Io	2,500	261,000
II.	Europa	2,100	415,000
III.	Ganymede	3,550	664,000
IV.	Callisto	2,960	1,167,000

Here, again, the largest body fills the centre of the field.

With Uranus, we have: —

No.	*Name.*	*Diameter in Miles.*	*Distance from Uranus in Miles.*
I.	Ariel	500	120,000
II.	Umbriel	400	167,000
III.	Titania	1,000	273,000
IV.	Oberon	800	365,000

The same relative agreement of position and mass!

Position of largest mass.

Now consider the probability that this coincident arrangement should be due to chance. The greater mass might be found either at the beginning, in the middle, or at the end of the line. Take, as starting point, that it is found to occupy the middle of the line in the Saturnian system. The chance, if chance arranged it, that it should occupy the like position in the solar system is one out of three, or two to one that it did not. That it should also do so in the Jovian system is $\frac{1}{3}$ of $\frac{1}{3}$, or eight to one against it. That furthermore the Uranian system should show the same is $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$. In other words, it is twenty-six to one that the largest satellite would not be found to occupy in all the same position. And it does. Twenty-six to one in betting is very much better than certainty odds.

Of second largest.

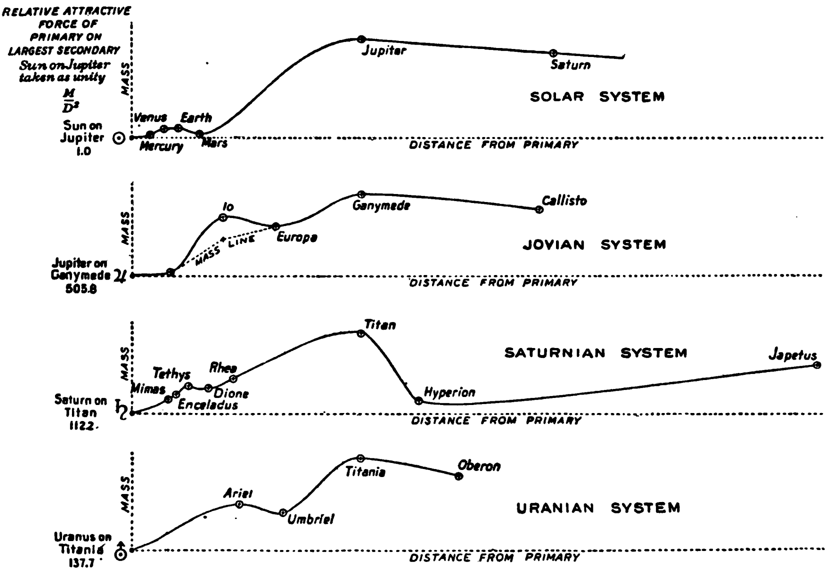
This is not all. Consider the four systems more carefully. It will be seen that the second largest mass is in each of them found outside the first and in three out of the four next to it. In the fourth, it comes next but one. Now the chances against this being accident are much greater than for the first coincidence; while the chance that the two chances should occur together as they do is the product of both. You will see that we are getting outside any chance in the matter at all, and have come face to face with some cause working to this end.

Of second maxima.

But we are by no means done with the analogies yet. If we construct a curve of positional sizes, we discover that it has two maxima, not one.

A second lies inside the first. In the solar system, our Earth occupies this place; in the Jovian, Io; in the Saturnian, Tethys; in the Uranian, Ariel.

Plotted in curves, the profiles of the four systems show a striking family resemblance, as can be seen from the diagram; and from what we have noted of the probabilities in the case, we cannot doubt that this betokens a law of system development.



POSITION OF MASSES IN SATELLITE SYSTEMS

FIG. XI.

Inclinations of orbits to planet's equator with increase of distance from planet.

A second point connected with the system is the relative inclinations of the orbits to the plane of the planet's equator. The inclinations to the planet's equator of the rings and of the several satellites proceeding outward are as follows: —

SATURNIAN SYSTEM.

	Inclination of Orbit to *Ecliptic.*			*Planet's Equator.*		
	°	′	″	°	′	″
Planet's equator . . .	28	10	22			
Rings	28	10	10	0	0	12
Mimas	28	10	10	0	0	12
Enceladus	28	10	10	0	0	12
Tethys	28	10	10	0	0	12
Dione	28	10	10	0	0	12
Rhea	28	10	10	0	0	12
Titan	27	38	49	0	31	33
Hyperion	27	4.8		1	5	34
Iapetus	18	28.3				

It thus appears that the inclinations of the planes of the orbits to the plane of the planet's equator increase as the distance from Saturn increases; furthermore, that the increase is regular. A smooth curve represents them all.

Now let us turn to the Jovian system.

Same in Jovian system.

The inner satellite, or Benjamin of the family, moves apparently in the plane of its primary's equator.

JOVIAN SYSTEM.

		Inclination of Orbit Plane to Planet's Equatorial Plane.
I.	Io	0° 0′ 0″
II.	Europa	0° 1′ 6″
III.	Ganymede	0° 5′ 3″
IV.	Callisto	0° 24′ 35″

Here, again, the inclinations increase as we go out, and the smooth curve representing them is,

INCLINATIONS OF SATELLITE ORBITS TO PRIMARY'S EQUATOR

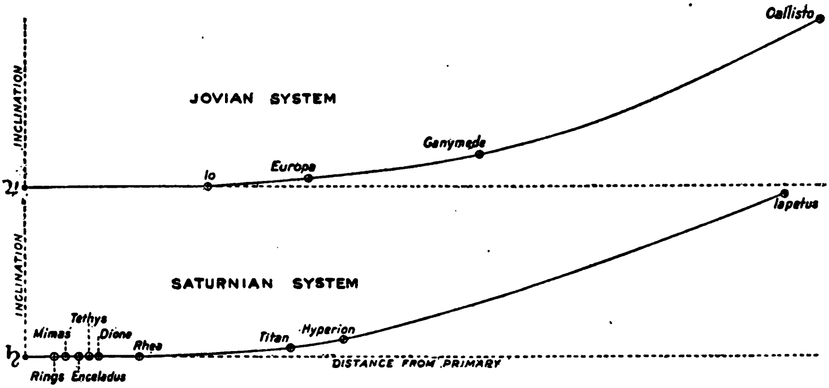


FIG. XII.

when reduced in scale, almost the counterpart of the Saturnian.

Clearly some force has operated to compel the satellites to travel in the planet's equatorial plane, and this force has emanated from the planet, since it grows less potent as one departs from him.

Force occasioning this due to planet.

What this force may be, we shall now proceed to ascertain.

Combination of rotations.

In order to make the action in the case, complicated at best, as understandable as possible, I shall begin by considering what causes the precession of the equinoxes, or that slow rotation of the pole of the Earth round the pole of the ecliptic.

Effect of pull on stationary spheroid.

Were the Earth a sphere, its axis would maintain an invariable position in the heavens, since any other body would act upon it as if all its matter were collected in its centre; but with a spheroid the case is different. We may consider the equatorial protuberance as a ring of matter fastened after the manner of a life-preserver around the Earth's waist. Now suppose the Earth tilted up from the line joining its centre and the centre of the attracting body. That body would tend to pull the nearer part of the ring down into its plane and the more distant portion of the same up into the same plane, and the result would be, if the Earth were not rotating, a swing round an axis at right angles to the line joining the centres of the two bodies, which would, after a few oscillations, bring the equatorial bulge to rest in the orbital plane of the outside body.

Upon rotating spheroid.

Now suppose the Earth to be rotating at the time the pull is applied; then the simultaneous rotation and pull entirely alters the problem. From being a statical, it becomes a kinematical one, and the outcome is utterly different from what we might expect. Instead of bringing the plane of the equator into the plane of the ecliptic, it swings the pole of the equator round the pole of the ecliptic in a direction at right angles to the pull, and opposite to the rotation, but without changing the inclination of the two planes permanently at all. If the axis be in such position that the pull is perpendicular to the rotation, no change of inclination, even temporarily, occurs. If the axis be so circumstanced that the pull is at any other angle to it, then the change of axis being always perpendicular to the pull, one component of the change rotates the axis as before, the other alters its inclination.

Precession and nutation.

Now if, as is the case with the attracting bodies of the solar system, the body which exerts the pull revolve about the other, either really or virtually, the axis will be presented to the force under varying angles. The axis will then alternately approach and recede from the pole of its small circle while going round it. But at the end of its orbital revolution it will come out again at the point on the celestial sphere from which it started. And this will happen whether the orbit be a circle or an ellipse. Even if the nodes of the ellipse or its line of apsides regress or progress, this will only postpone the reëntrance of the curve into itself to the time when the nodes or perihelia, or both, shall have completed their revolutions.

No permanent change in axis.

No permanent change in the inclination of the axis to the orbit can ever result from the pull of a second body upon the first's equatorial bulge.

Since action and reaction are equal and opposite, the equatorial protuberance is equally impotent to make the satellite travel permanently in its plane.

Same analytically.

This appears also analytically in the expressions for the effect produced in the line of nodes and the effect upon the inclination, the former having in addition to its periodic terms a term which increases with the time, while the latter has no such term.

Error in Young.

It may be worth pointing out here an error which has crept into Young's excellent text-books, in which he states that "Laplace and Tisserand have shown that the equatorial protuberance of a planet, due to its axial rotation, compels a near satellite to move nearly in the equatorial plane." Neither Laplace nor Tisserand has ever shown this or ever could.

Laplace and Tisserand.

What they did show was that the expression for the perturbative action of the equatorial bulge of a planet denotes that the inclination of the satellite to the plane of the planet's equator remains constant under the action of that force. Now this could be true, either because the force had a restraining effect to this end, or because it *had no effect upon the inclination at all.* Laplace jumped to the conclusion that the first was the case, for he tells us, apropos of Saturn and his next to outer satellite, that we see "that Saturn's action can retain this satellite in very nearly the same plane; and much more so those satellites which are inferior to it, as well as the rings."[1] He made the mistake of *post hoc ergo propter hoc.* Tisserand is more guarded when he says: "Ainsi, l'inclinaison de l'orbite d'un satellite sur l'anneau demeure constante et toujours très petite si elle l'a été seulement à un moment donné." This is so; but it is true, not because the force has an effect upon the inclination, but precisely *because it has none.*

The spherical ellipse found by Tisserand, t. iv., ch. vi., to represent the change of inclination in the case of the satellites of Saturn, is the curve of the combined precessions due to each of the perturbing forces, the equatorial protuberance, the ring, the sun, and the other satellites.

Effect of tidal crest.

Impotent on the inclination as the equatorial protuberance is, there is another protuberance which is not so impotent. For consider what effect the tide-raising force of an outside body would have upon the plastic matter of another rotating in a plane tilted to the orbital plane of the first. As we saw in Chapter II., the effect would be to raise two bosses or ansæ in the equatorial

[1] See Laplace, Book IV., § 26. At the time, Hyperion was undiscovered and the "next to outer satellite" in consequence different.

plane of the rotating planet, one preceding the position of the tide-raising body, the other diametrically opposite.

The action of these ansæ upon the attracting body would be analogous to, but in one vital respect different from, that of an equatorial protuberance. Like that, they would tend to alter the position of the axis of rotation at right angles to the pull upon them, but the pull being always backward the axis is constantly solicited forwards toward the attracting body. Consequently the axis of rotation, while rotating round the axis of the orbit, would generally seek the satellite. For the force here, when the axes are perpendicular, is at its maximum. The axis, therefore, continues to tend toward the orbital plane.

Same analytically.

Analytically, in this case, unlike that of an equatorial bulge due to axial rotation, the expression for the change of inclination contains a term dependent on the time and increasing with it.

This term causes the inclination of the equatorial to the orbital plane to diminish until the axis of rotation lies in the plane of the orbit.

Inclination of satellite's orbital planes to planet's equator.

The tidal force varies as $\frac{2mr}{d^3}$, approx., and its work for any given time as $\frac{4m^2r^2}{d^6}$, approx.

It should therefore be much more potent upon a near satellite than upon a far one, and we should expect the line expressing the action to prove a curve concave to the axis of x, when the bodies acted on are not too dissimilar in size. Such is precisely the opposite of the curve the diagrams present.

Nevertheless tidal action is probably the cause of the law of inclinations shown in the orbits of satellites to the equatorial plane of their primary. But it would seem to imply that the farther ones were given off first, and very much the first.

V

JUPITER AND HIS COMETS

Jupiter exemplifies chaos.

CHAOS describes Jupiter at present; the seething something between sun and world. The planet is either a sun in its senility or an earth in its babyhood, as you are pleased to regard it. For the one state passes by process of development into the other.

A semi-sun.

Viewed as a sun, it lacks little except light; viewed as a world, it wants everything except that lack of luminosity. It is, as Virgil described another giant, *informe ingens cui lumen ademptum.* Its density is almost exactly that of the Sun itself. Either, therefore, its bulk is chiefly atmosphere round a kernel of planet, which is Professor Darwin's conclusion, or its smaller mass is offset by its lesser heat, causing a like condensation of the two globes. On the latter supposition, though not luminous, it is still hot. This would bear out and confirm the inference, from the brick-red color between its belts, that its surface is at a red heat.

Almost precisely the same is true of Saturn; the body of that planet, too, being a faint cherry red. Jupiter, however, we see much the better of the two, and we may describe it as typifying both.

Both are bulky; their masses to their volumes being such that their mean densities are respectively somewhat greater (1.28 %) and somewhat less than water (.69 %). Both are in rapid rotation; particles on their equators traveling with speeds comparable with their orbital velocities. Both, in consequence, are strikingly flattened into oblate spheroids whose elliptic curves instantly strike the eye. In the disks of both we look only upon atmosphere and cloud. Lack of solidity, speed of self-movement, cloudy condition, are all so many signs of — youth. In relative — if not in absolute — age, both planets are still very young.

Semi-suns in several senses, the two planets are three-quarters way in their journey from nebula to world. In their traits both more closely resemble the Sun than the Earth. Indeed, with the trifling exception of not shining, the disk of Jupiter or of Saturn bears a very remarkable analogy to the solar.

Ruddy glow.

In a large telescope and in good seeing, Jupiter is a color-picture as beautiful as it is marked. A deep pink flush suffuses the planet's equatorial regions. It probably betokens the parts of the true surface that are laid bare. For that the color is due to the selective absorption of the higher regions of the planet's air is negatived by the spectroscope, which shows dark bands in the red.

Rotation. In spite of its enormous bulk, Jupiter turns on its axis with such speed that its figure is flattened by $\frac{1}{15.5}$. Its mean time of rotation is $9^h\ 55^m$. We are forced to say its mean time, not because the markings cannot be accurately timed, nor because of any change in the planet's moment of momentum, but because the planet does not rotate as a whole. Different parts of it go round at different rates. Speaking broadly, the nearer the equator the greater the speed. Between the equator and latitude 30° there is a difference of six minutes in the rotation period. But the several belts have each its own period, and this does not always accord with the latitude. In addition, particular spots on the same longitude have particular spins, and pass by each other at speeds from seven miles to four hundred miles an hour. White markings travel faster than dark markings close beside them. Thus the white masses around the great red spot drift by it. The spot itself has changed its rate by six seconds in as many years. It is pretty evident that Jupiter is chaotic.

The same is the case with Saturn. Stanley Williams, in 1893, found for the Saturnian regions between 6° N. and 2° S., $10^h\ 13^m$, and for those between 17° N. and 27° N., $10^h\ 15^m$. Not only did latitudes differ in rate, but different longitudes went each at its own pace.

Rotation of Saturn.

Something similar is true of the Sun. At the solar equator the spin is swifter than on either side of it; and the rate decreases steadily from the equator towards the poles. Spots near the equator go round in 25 days (25.23 days), spots in latitude 30° in 26½ days, in latitude 40°, 27 days, while in latitude 45° they take fully two days longer than in 0°. Now Willsing and Professor Sampson, of Durham University, have shown that such a state of things should result in the process of condensing from nebula to star. In the nebula, if the density varied from place to place, which, on the doctrine of chances, would certainly be the case, the several parts would revolve round their common centre of gravity at various rates. As the nebula condensed, such parts as held together would tend to equalize their individual motions through friction, until a common rotation was brought about. But this would consume a long time; in the mean while, the equatorial parts would outstrip the others. In the midst of the

Sun's rotation.

equalizing process the Sun, Jupiter, and Saturn now seem to be.

Jupiter has cloud layers.

Jupiter, however, has progressed beyond the Sun, in that the outer layers of his substance have cooled down enough to condense into cloud, due, possibly, to the planet's smaller mass. On the surface of the Sun things are still kept largely uniform by the terrific heat, and the slower rotation lets us perceive no latitudinal layers. On the contrary, Jupiter's disk is striated with belts of various tone and tint, according almost exactly to the parallels; while the albedo, or relative brightness of the disk, 62 per cent. of absolute whiteness, indicates that most of it is cloud.

Jupiter's clouds self-raised, not Sun-raised.

These clouds are quite unlike our terrestrial ones. Jupiter's clouds are not Sun-raised, but self-raised condensations. On the one hand, the Sun's action there, only $\frac{1}{27}$ of what it is here, is impotent to produce the effect we see; on the other, the cloud zones show a persistence quite disregardant of the Sun. They are not ephemeral like ours, but long-lived, lasting for weeks, months, and even years. They must, therefore, be Jove-caused.

Disk darkens toward limb.

In another feature Jupiter resembles the Sun. Its disk darkens to the limb. None of the smaller planets do this. The only thing capable of producing such effect is a layer of atmosphere surrounding the disk of considerable depth. Jupiter's atmosphere is dense, and the absorption to which a ray of light would be subjected in passing in from the Sun and then out to us would increase from centre to circumference, and thus dim the edges of the disk.

Comets associated with Jupiter.

Jupiter has two families of bodies connected with him; one an own one of satellites, the other an adopted one of comets. With his satellites we made acquaintance in the last chapter; we must now be introduced to his comets.

Thirty-two comets circle near the planet and agree in the following distinguishing characteristics:—

1. Their aphelia hug Jupiter's orbit.
2. Their ascending nodes occur close to it.
3. Their motion is direct.

Association suggests capture.

At some time in the past, therefore, each of these comets must have passed close to Jupiter, the comet and the planet chancing to arrive together at the node. At that epoch the comet must have suffered great disturbance at the hands of the planet, and its previous orbit have been radically changed.

D'Alembert, accordingly, suggested that Jupiter had captured these comets, and Laplace

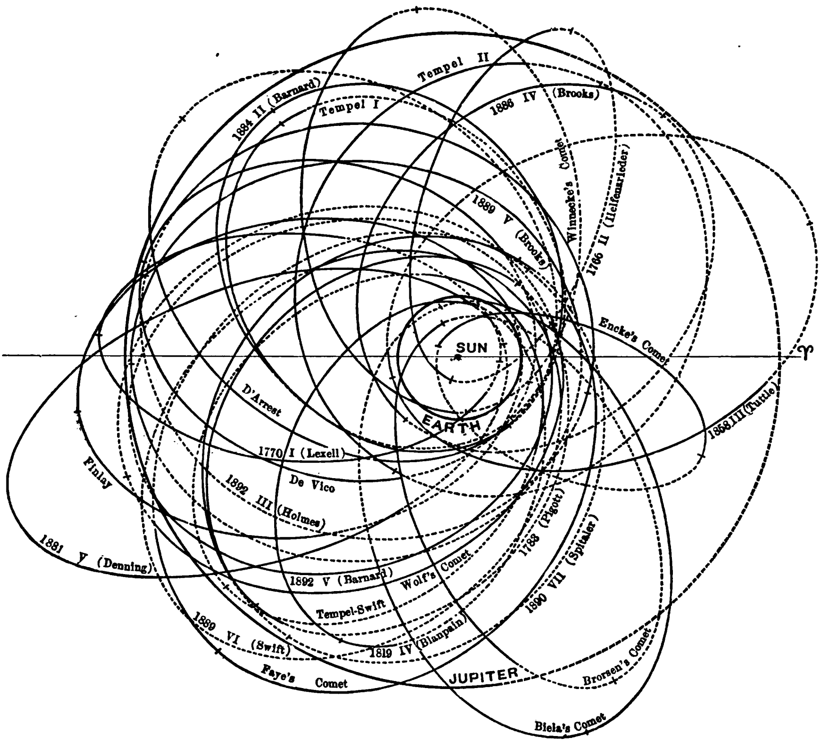


FIG. XIII. JUPITER'S FAMILY OF COMETS.

extended the idea; but to Professor H. A. Newton we owe the most important research in the matter. In two striking memoirs (1878 and 1893), he showed that Jupiter was quite capable of such capture; but he started with the assumption that comets were not denizens of the Sun's domain, so he considered only parabolic comets.

Comets all belong to the solar system.

We now know that all comets probably that man has ever seen are part and parcel of the Sun's retinue. They do not come to us from outer space, but are stable, if erratic, members of the solar system. In the light of this fact, we may profitably reconsider the subject.

Jupiter's sphere of influence.

Picture a comet, coming in to the Sun from space, to pass close to the planet in its journey. Within a certain distance of Jupiter, the planet's pull becomes so great that it is mechanically more exact to regard the comet as obeying Jupiter and perturbed by the Sun; and if the approach be very close, we may neglect in a first approximation the Sun's effect during the passage. This region is called Jupiter's sphere of influence, and is of the general shape of an ellipsoid, whose longest diameter follows the planet's path. The mean radius of the ellipsoid is three tenths of the Earth's orbit, no inconsiderable distance, and the extreme radii differ as 1 to 1.19.

Relative orbit about planet an hyperbola.

As the comet is traveling, when it enters the planet's sphere of influence, with Sun-imposed velocity, its speed, even if the orbit be elliptic of small major axis, will exceed what Jupiter could cause. It will, in general, approach Jupiter with

Jovian hyperbolic velocity, and its relative orbit about the planet will be an hyperbola. Jupiter, therefore, cannot completely possess itself of the comet.

Comet accelerated or retarded according as it passes behind or before the planet.

The general equation of the relative motion I shall not bother you with. But certain deductions from it I think you will find of interest. In the first place, it appears that the comet will be accelerated or retarded, according as it passes behind or in front of the planet. This may be seen directly from the consideration that if it pass in front of the planet, it accelerates the latter, and since action and reaction are equal and opposite, it must itself be retarded; contrariwise, if it pass behind the planet.

Parabola made into ellipse by retardation.

Suppose now the comet to have been pursuing a parabolic path before the encounter; then the least retardation will make of its orbit an ellipse; for whether a body move in an ellipse, a parabola, or an hyperbola is a question simply of its speed at a given distance, shown by the well-known equation, —

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right).$$

Into hyperbola by acceleration.

Similarly, the least acceleration will throw it into an hyperbola, and it will pass out of the solar system, never to return.

For an original elliptic orbit, this is not necessarily the case. A comet pursuing such a path may have its velocity increased and yet not pass out of the system. In many cases, however, it would so result, and we can thus perceive how comets might come to us from other systems from purely internal forces there.

Jupiter's maximum effect in shortening comet's major axis.

The maximum effect in retarding the comet's motion occurs when the comet approaches Jupiter in such a direction and with such a relative speed as to be turned back upon the planet's path, and to leave the planet in the direction of the planet's quit, with a relative speed equaling the planet's own. It is then left stock-still to fall into the Sun.

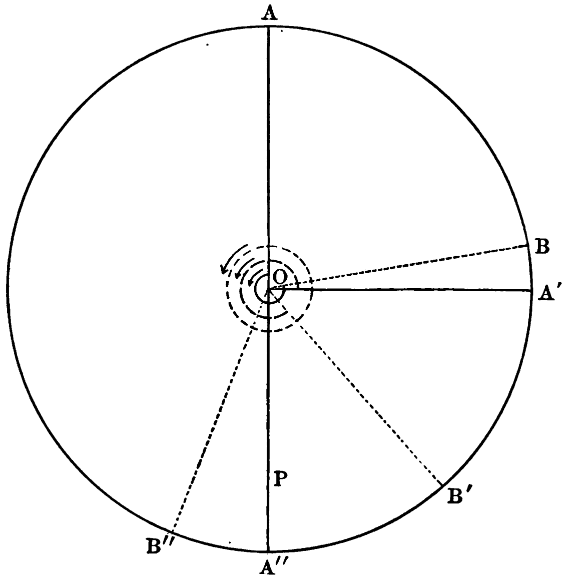
Jupiter's absolute maximum effect.

Jupiter can do more than this. Though to leave a comet stock-still to drop into the Sun, thus shortening the major axis to one half its own, is its maximum effect in the way of contracting the orbit, its power over the comet exceeds such limit. The planet can actually prevent a comet bound round the Sun from attaining its object. It can cause the comet to make itself in place of the Sun the goal of its pilgrimage, and sweeping round the planet, to go back into space without visiting the Sun at all.

Consider the hyperbola the planet causes the comet to describe. What the planet does is to

Jupiter's bulk limits his power.

swing the incoming asymptote of this hyperbola through a certain angle. Clearly, the closer the perijove of the relative orbit, the greater this angle



The comet's direction may be turned from OA to OB; or OA′ to OB′; or OA″ to OB″, according as it approaches along OA, OA′, or OA″, P being the planet.

FIG. XIV. RELATIVE ORBITS.

of swing, as the planet gets a greater pull upon the particle. If the comet were not coming too fast and Jupiter's own body did not get in the way, the comet could be turned straight back whence it came. Practically, Jupiter's bulk does get in its way, and the limit of the planet's power lies below such direct reversal; nevertheless, it is sufficient in many positions to cause the comet to sweep round and dart away from the Sun with a speed such as to carry it beyond the Sun's control.

The planet's greatest effect in turning the comet is shown in three different conditions of approach. The comet enters along the unbroken lines and leaves by the broken ones.

Deflective power.

You will notice that Jupiter's power is solely one of deflection. He cannot vie with the Sun directly in a tug of war; but he can deflect the comet and thus use the very speed imparted by the Sun against the Sun's attraction. It is like the Japanese *jiu-jitsu*, or scientific wrestling, of which the art consists in so adroitly turning another's strength against himself as to make the man's own momentum cause his fall.

Triangle of velocities.

Considering the case in this wise, we shall have the key to all of Jupiter's control. Form a triangle of velocities, of which the one side shall represent Jupiter's motion in amount and direction, a second the comet's, and the third the relative motion of the one body about the other; then draw a circle with the last for radius from the meeting-point of the planet's and comet's true motions, and join any other point of it to its centre. This second radius will represent the outgoing asymptote of the relative orbit, according to the planet's pull, while the line joining its peripheral end to Jupiter will be the comet's subsequent motion in amount and direction.

From this you will perceive that the comet's subsequent career depends upon the actual speed with which, the angle under which, and the nearness to which, it approaches the planet. If it creep upon the planet from behind, it is more likely to be captured than if it meet it head on; and if it be traveling slowly, it is more likely to be caught than if it were going fast.

Direct orbits made retrograde.

Any one of many things may happen. If it pass behind the planet, its actual speed is increased, and either it is sent clean out of the system, or it is at least put farther from capture than before. If it pass before the planet and in such a way that its relative speed about the planet exceeds the planet's own motion, and it is turned round through a sufficient angle, it may, from a previously direct path about the Sun, be diverted into a retrograde one. In this case, it will commonly have a small velocity after the encounter and retrograde in a small ellipse.

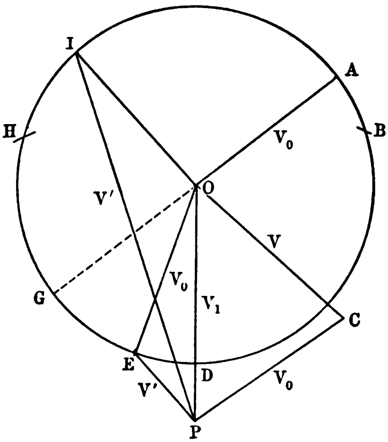


FIG. XV. ACTION OF JUPITER.

V represents in amount and direction the comet's actual velocity in space. V_1 similarly denotes that of the planet, the two bodies meeting one another under the angle VOV_1. V_0 will then represent the relative motion, in amount and direction, with which the comet approaches the planet.

The action of the planet is to turn the relative motion of the comet through an angle, say AOE, OA representing the in-coming asymptote of the relative orbit, OE the out-going one. EP or V' will then represent the absolute motion in space of the comet after the encounter. Similarly, if the comet passed behind the planet and was turned through the angle AOI, PI would be the new absolute velocity of the comet on leaving the planet.

The critical angle.

If, however, its entering speed and approaching angle, which we will call ω, are such that $\cos \omega < \frac{1}{2} \cdot \frac{v}{v_1}$, where v is its actual velocity, v_1 that of the planet; then its relative velocity, v_0, can never be greater than v_1, and the resulting orbit never can become retrograde. This angle we will call the critical angle, and designate it by the symbol χ.

Now ω we can calculate for each of the comets of Jupiter's family from their known present paths. Furthermore, since Jupiter's only effect is to swing the outgoing asymptote of the relative orbit round, v_0 can never be changed, and the future possible values of ω have a superior limit ω', which they can never pass. This also we can calculate. Doing this, and calculating also the value of χ for each comet, we find the table on the opposite page.

ω and ω' both less than χ in all comets of Jupiter's family.

From the table, it appears that in every one of the comets of Jupiter's family, ω is within the critical angle.

Furthermore, that ω', the maximum value which ω may attain under the perturbative effect of the planet, owing to the swing of the asymptotes of the hyperbolic relative orbit of the planet, is also always within χ.

PLATE XI. JUPITER'S COMET FAMILY.

Galle's Cat. No.	Discoverer.	Inclination of orbit to ecliptic.	Major axis.	Peri-helion.	Aphel-ion.	α Angle between comet's path and Jupiter.	ω' Maximum value possible for ω.	χ Critical angle.	σ Angle of stability.
						°	°	°	°
64	. . . .	+ 2.9	6.14	1.1	5.0				
85	. . . .	+ 1.9	6.18	.9	5.3	18.7	30.1	73.6	
100	Helfenzrieder* .	+ 8.0	5.88	.4	5.5	37.9	43.4	76.1	
102	Lexell* . . .	+ 1.6	6.35	.7	5.7	36.6	38.9	72.4	8.8
105	Biela	+17.1	7.10	1.0	6.1	41.3	41.4	68.5	21.1
113	Pigott* . . .	+45.1	6.52	1.5	5.0				
117	Encke	+13.6	4.42	.3	4.1				
158	Winnecke . .	+10.7	6.32	.8	5.5	32.1	36.0	72.7	6.6
159	Blanpain* . .	+ 9.0	5.71	.9	4.8				
200	Faye	+11.4	7.61	1.7	5.9	28.9	29.0	66.6	23.1
201	De Vico . . .	+ 2.9	6.20	1.2	5.0				
210	Brorsen . . .	+30.9	6.29	.7	5.6	45.7	46.0	73.0	0.0
230	D'Arrest . . .	+14.2	6.89	1.2	5.7	31.8	33.2	69.5	19.7
259	Tuttle* . . .	+19.5	7.06	1.1	5.9	37.8	38.0	68.7	21.0
293	Tempel . . .	+ 6.4	6.37	1.6	4.8				
300	Tempel-Swift .	+ 5.5	6.20	1.1	5.1				
311	Tempel . . .	+12.7	5.98	1.3	4.7				
348	. . . .	+ 6.8	8.43	.7	7.7	55.3	61.0	64.0	24.3
358	Barnard* . . .	+ 5.4	6.11	1.3	4.8				
359	Wolf	+25.3	7.15	1.6	5.5	32.8	33.0		
368	Brooks* . . .	+12.9	6.32	1.3	5.0				
371	Finlay	+ 3.0	7.08	1.0	6.1	38.0	38.0	68.3	21.5
388	Brooks . . .	+ 6.1	7.37	1.9	5.4	15.8	20.0	68.7	21.2
389	. . . .	+10.2	8.36	1.4	7.0	41.7	43.0	67.5	22.5
396	Spitaler* . . .	+12.8	6.87	1.8	5.1			64.2	24.4
404	Holmes . . .	+20.8	7.23	2.1	5.1				
406	Barnard* . . .	+31.2	6.78	1.4	5.4	34.4	36.0	70.1	18.5
412	. . . .	+ 5.6	7.80	1.1	6.6	41.6	42.0	65.9	23.6
417	. . . .	+ 3.0	7.36	1.2	6.2	33.6	34.0	67.5	22.5
424	. . . .	+11.3	7.00	1.5	5.5	24.6	27.0	69.0	20.5
426	. . . .	+13.7	6.92	1.1	5.8	34.4	35.0	69.4	19.9
447	. . . .	+29.9	7.23	.9	6.3	49.5	50.0	68.0	22.0
Average	. . . .	12.9	6.73	1.2					

Therefore, of the comets of Jupiter's comet-family, not only is none now retrograde, but none can ever become so unless some other body interfere with it.

ω nearly equals ω' in almost all cases.

A singular coincidence characterizes the values of ω and ω'. In all but two cases, ω nearly equals ω', as if for some reason ω were always trying to attain this maximum as a condition of stable equilibrium. In ten cases out of twenty, or in one half of the whole, the approach is within less than $\frac{1}{2}°$.

Potential relative velocity remains unchanged.

It is to be noticed that in orbits potentially retrograde, the potential direct velocity is also greatest; so that both on the score of retrogradation and of greater direct velocity, comets pursuing such orbits are more subject to expulsion.

Comets of high potential relative velocity the first to disappear.

In course of time, comets possessing a high potential velocity must be weeded out of the system; for, sooner or later, they must meet the planet under conditions of approach which convert their high potential velocity into an actual one. This will happen the sooner for comets in proportion to their velocity possibilities. It therefore will occur more speedily for originally parabolic comets than for elliptic ones of short period; but it will require some time even for them.

Either, then, Jupiter's present comet family has been of very slow growth, and each comet remains for a long time in the family, or it is made up only of short-period comets drawn from the immediate neighborhood.

Comets ephemeral things.

Now, comets appear to be ephemeral things, being easily disintegrated into meteor swarms, and never abiding long in one stay. Thus the latter supposition seems on the face of it the more likely. We may conclude provisionally that Jupiter's comet family came from the neighborhood.

Jupiter has cleared his neighborhood.

It is certain that Jupiter has swept his neighborhood of such comets as do not fulfill the criterion of the angle χ; that is, of all the comets actually or potentially retrograde. If we consider the comet aphelia of short-period comets, we shall notice that they are clustered about the path of Jupiter and the path of Saturn, thinning out to a neutral ground between, where there are none. Two thirds way from Jupiter's orbit to Saturn's, space is clear of them, the centre of the gap falling at 8.4 astronomical units from the Sun.

Let us consider the mean comet; that is, a comet having the mean inclination of parabolic comets, the mean perihelion distance of the comets of Jupiter's family, — such being the distance most likely to disclose them to us, — and let this mean comet have successively aphelion distances from Jupiter's orbit to Saturn's.

Mean inclination of comets: theoretical.

The mean inclination we may take either as the mean of comets coming to us from all parts of space indifferently or as the mean of such parabolic comets as have actually been observed.

If we suppose the inclinations of the cometary orbits to be equally distributed through space, then the poles of the orbits will likewise be strewn uniformly over the celestial sphere. If α be the angle made by a pole with the pole of the ecliptic, the mean inclination of the poles can be found by multiplying the number of poles at any inclination, which is as the strip of surface yielding it, by that inclination, and then dividing the integral of this for the whole sphere by the surface of the sphere. The strip of surface at any inclination α is $2\pi r^2 \sin \alpha . d\alpha$. Whence the average inclination in radians is

$$\frac{\int_0^{\pi} 2\pi r^2 \sin\alpha . \alpha . d\alpha}{\int_0^{\pi} 2\pi r^2 \sin\alpha . d\alpha} = 1$$

or $$i = 57^\circ.3.$$

Mean inclination observed.

The second mean inclination or actual mean of all the parabolic orbits observed is $i = 52^\circ.4$.

It is worthy of notice how near the two are, showing that the parabolic comets come to us, practically, indifferently from all parts of space.

Calculating ω and χ for the successive aphelia, we find that, on the first supposition, ω passes χ at 8.4 astro. units; on the second, at 8.75 ditto.

It is Jupiter, then, that has swept this space of comets.

Family larger than we see.

Only a small fraction of Jupiter's comet family can ever come within our ken; for any comet whose perihelion lay outside of two astronomical units must, perforce, escape recognition. Invisibility would be caused both by the comet's distance from us and by its distance from the Sun, for the commotion set up in these bodies, as they near the Sun, is chiefly responsible for the display they make.

The family undoubtedly consists of many more comets with greater perihelion distance.

Jupiter is not the only planet that has a comet-family. All the large planets have the like. Saturn has a family of two, Uranus also of two, Neptune of six; and the spaces between these planets are clear of comet aphelia; the gaps prove the action.

Nor does the action, apparently, stop there. Plotting the aphelia of all the comets that have been observed, we find, as we go out from the Sun, clusters of them at first, representing, respectively, Jupiter's, Saturn's, Uranus', and Nep-

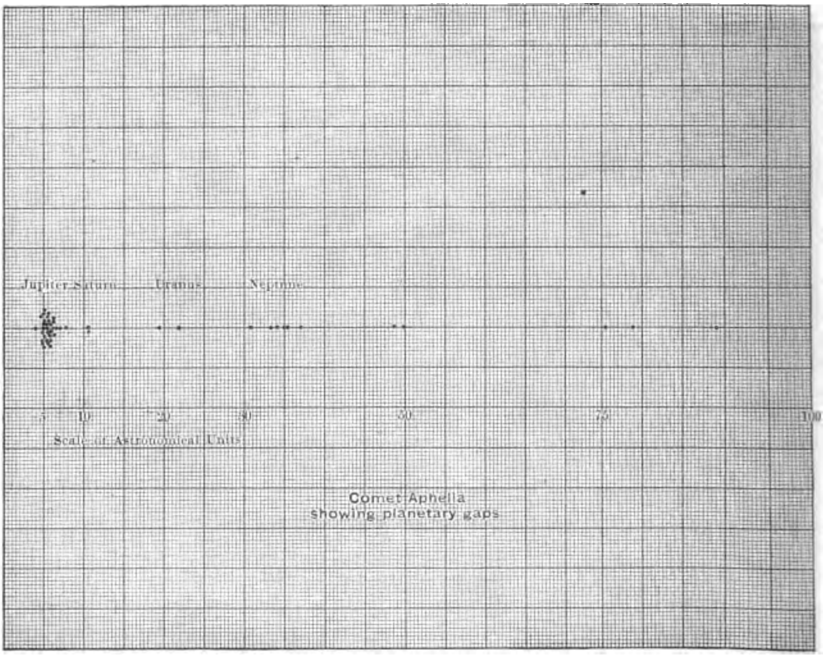


Fig. XVI. Comet Aphelia.

tune's family; but the clusters do not stop with Neptune. Beyond that planet is a gap, and then at 49 and 50 astronomical units we find two more aphelia, and then nothing again till we reach 75 units out.

This can hardly be accident; and if not chance, it means a planet out there as yet unseen by man, but certain sometime to be detected and added to the others. Thus not only are comets a part of our system now recognized, but they act as finger-posts to planets not yet known.

We have thus examined the case of an old planet, — Mercury; of a middle-aged one, — Mars; of a youthful one, — Jupiter; and we have ended by envisaging the yet unchristened.

VI

COSMOGONY

Present the outcome of the past.

After the present, the past. The forces that we have found to be moulding the system to-day must be those that fashioned it earlier. Given, therefore, the condition at the moment, if we apply to it the forces now at work reversed, we shall get the condition that was.

Similarly, we can cast its horoscope for the future, — by Taylor's theorem.

Unfortunately, the problem is so complicated that no solution, even approximately satisfactory, has yet been obtained; but that the mystery baffles us renders it all the more fascinating.

Striking relations in the solar system.

In the solar system, as we find it to-day, are several remarkable congruities which are quite independent of gravitation, and bespeak a cause.

I. The central body is much larger than its attendants.

II. The planets move in orbits nearly circular.

III. They travel nearly in one plane.

IV. And in the same sense (direction).

As for the planets themselves —

V. Their planes of rotation nearly coincide with their orbital planes (except Uranus and Neptune).

VI. They rotate also in the same direction that they revolve, counter-clockwise, all of them (except Uranus and Neptune).

VII. Their satellites revolve nearly in the planes of their primaries' equators (so far as we can see).

VIII. And in the same direction.

IX. They rotate in the same plane (so far as we can see).

X. In the same direction (so far as we can see).

Kant's nebular hypothesis.

Immanuel Kant was the first to suggest something approaching a rational explanation of this very curious and elegant state of things. He made the error, however, of supposing that rotation of the whole could be produced by collisions of its parts; but no moment of momentum can be caused by the interaction of parts of a system, since internal forces occur in pairs and their moments round any line are equal and opposite. We will consider this in detail a little further on. Laplace, who appears not to have known of Kant's writing, himself some years later developed a somewhat similar theory, but with more mathematical foundation. He assumed an original rotation and got the credit for the nebular hypothesis. He had a faculty of getting credit for things which was only second to his ability.

Laplace's nebular hypothesis.

To account for so orderly an arrangement Laplace supposed:—

a. That the matter now composing our solar system was once in the form of a nebula.

b. That this original nebula was very hot, a fire-mist.

c. That it possessed initially a slow rotation.

d. That as it contracted under its own gravity and thus, from the principle of conservation of moment of momentum, rotated faster as it shrank, it rotated always like a solid body with the same angular velocity throughout, until its outer portions, which went the fastest, came to go so fast that the centrifugal tendency overcame the centripetal force and they were left behind as a ring.

e. That this ring revolved as a whole until it broke, rolled back upon itself and made a planet; the outer parts of the ring having the swiftest motions, the resulting planet rotated in the same sense that it revolved.

f. The planet thus formed gave birth in like manner to its satellite system.

Physical error in Laplace's hypothesis.

The prestige of Laplace gave this explanation a mental momentum which has carried conviction nearly to the present day. But it is erroneous for all that, nor can it be made to work by any additions or slight alterations as some text-books will tell you. For it was founded on what it has now foundered on: one fundamental mistake. Laplace assumed that his nebula would revolve, as he saw the air around the Earth to revolve, of a piece. But he forgot that friction due to the pressure alone produces this, and that in particles moving freely no pressure exists. Under the pull of a central mass each layer of the nebula would revolve at its own appropriate rate, or as $r^{-\frac{1}{2}}$. So that his beautiful explanation of the agreement in direction of the rotations and the revolutions — the vital point of the theory — falls to the ground.

Faye's nebular hypothesis.

Faye first definitely pointed out this fatal fallacy in Laplace's hypothesis in 1886, in his "Origine du Monde," in which, after reviewing the previous history of the subject, he brought forward a new theory of his own, both elegant and ingenious.

He begins by assuming a nebulous mass of particles, roughly uniform throughout, but with local condensations. He supposes this nebula cold, for the heat can be trusted to come of itself. With uniform density throughout, the speed of rotation would also be uniform, thus giving the same result that Laplace got, but for a very different reason. In a spherical mass of matter of uniform density, a particle at any point is attracted only

Force originally as δr.

by the sphere within it. It is therefore pulled by the force $\frac{m}{r^2} = \frac{\delta r^3}{r^2} = \delta r$, where δ is the density.

Since the force is thus linear it may be resolved into two harmonic motions and becomes motion in an ellipse with the acceleration directed to the centre, or elliptic harmonic motion whose equation is expressed in vector coördinates:—

$$\text{whence} \begin{cases} \rho = a\cos(nt+e) + b\sin(nt+e), \\ \rho' = -n[a\sin(nt+e) - b\cos(nt+e)], \\ \rho'' = -n^2[a\cos(nt+e) + b\sin(nt+e)] = -n^2\rho. \end{cases}$$

The form of the ellipse depends upon the amount and direction of the initial velocity of the particle.

This equation shows, first, that the period of rotation is the same for all the particles; and second, that the angular speed in such different nebulæ is as the square root of their densities.

Subsequently as $\frac{m}{r^2}$.

When the mass has practically collected in the centre, the force is $\frac{m}{r^2}$, or the ordinary law of gravitation, giving elliptic motion with acceleration directed to the focus, or elliptic motion *par excellence.*

At any intermediate stage of the process he supposes the force to be represented by $f = \alpha r + \frac{\beta}{r^2}$, α gradually dying out and β increasing as centralization goes on.

Planets given off under the first state of things would rotate in the same direction in which they revolved; under the last in the opposite way. He, therefore, supposes the terrestrial planets to be the older; the outer planets the younger members of the system. His theory makes the order of birth the exact contrary of Laplace's.

More recently Lieutenant-Colonel R. du Ligondés[1] has evolved another cosmogony. Ligondés's general theory is ingenious, but to me not convincing. His first point is unqualifiedly good. He starts out by calling attention to the evidence offered by the moment of momentum of the solar system upon the early history of that system. He shows that to produce a single star system like ours, the original motions of the several parts of the nebula must have been nearly balanced, the plus motions almost canceling the minus ones.

Moment of momentum.

It now becomes of interest for us to consider this question. Conservation of moment of momentum is as fundamental in mechanics as the conservation of energy. The momentum of a body is its mass into its velocity, and the moment of momentum is this mass-velocity multiplied by the perpendicular upon its direction from the point

[1] *Formation Mécanique du Système du Monde*, Gauthier-Villars et Fils, Paris, 1897.

or line around which the moment is taken. The moment of momentum is thus twice the area swept out by the moving body about the fixed one in unit time.

When two bodies collide, the amount of motion is not changed. This truth is the result of experiment, and was first determined by Newton. If the two are perfectly inelastic, they move on after the collision as one mass with a loss of kinetic energy. If perfectly elastic, they rebound in such a manner that not only the amount of motion, but the kinetic energy remains unchanged. Now probably no bodies are perfectly inelastic, just as no bodies are perfectly elastic. In the case, therefore, of the bodies in nature, while the amount of motion is never altered, a part of the kinetic energy is lost by the shock. It is transformed into heat energy.

Moment of momentum constant.

Now the moment of a velocity, and therefore of a momentum, clearly remains constant when unacted upon by any force, for its direction continues the same, and a perpendicular upon it from any point measures out the same area in the same time, as the perpendicular, too, is constant.

The like is true, if it be acted upon by a force constantly directed to the same point; for in that case the force can generate no velocity except along the perpendicular upon the line which represents the body's momentum, and therefore cannot change the area swept out.

When two bodies collide, therefore, they each bring an eternal definite amount of motion to the collision; this amount is unaffected by the shock.

Nor can the mutual attraction of the two bodies themselves alter it; for, since a force is measured by the amount of velocity it can generate in a given time, the velocities generated must be as the opposite masses, and therefore the momentum produced in each be the same.

Let m and m_1 be the masses.

Then $f_m t = m_1 v_m,$

and $f_{m_1} t = m v_{m_1},$

and $\frac{v_m}{v_{m_1}} = \frac{m}{m_1};$

whence $m_1 v_m = m v_{m_1}$

or $\text{Aa} = -\text{Bb}$

where $\text{Aa} = \frac{v_m}{m}$ and $\text{Bb} = \frac{v_{m_1}}{m_1}$

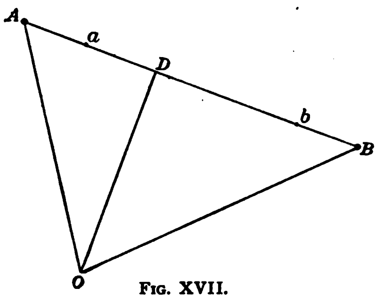


FIG. XVII.

Moreover, it is directed in both cases along the same line. Whence its moment in the two cases about any point is the same in amount, the perpendicular from the point being common to both, but opposite in direction. The two moments thus destroy one another. From which we see that the internal forces of a system are unable to change the moment of momentum of the system. Similarly they are incapable of having created it to begin with.

The present moment of momentum of the solar system can be calculated. It is found to be nearly the least possible. It must, therefore, always have been so. It was predestined by internal motions to make a single star.

So far, he is admirable, but from this point I lose him; I cannot see the cogency of all his succeeding steps. They lead him to the conclusion that everything is as it should be, and incidentally that Jupiter or Neptune is the oldest planet, Uranus the next, then Saturn, Mars, the Earth, Venus, and Mercury. The importance of the order will appear shortly.

Trowbridge's explanation of direct and retrograde motion.

With regard to the retrograde rotations of the outer planets and the direct rotations of the inner ones, Trowbridge suggested that uniform density, or a density increasing toward the centre, would account for it. Suppose, first, the density uniform, or nearly so. Then the inner parts of the mass that went to form the planet would be traveling fastest, and their momentum would prevail over that of the outer particles and give a retrograde rotation to the whole. Suppose, however, that the density increased toward the inner side of the mass. Then the centre of inertia would be so far shifted toward the inner edge, say to *N*, that the sum of the moments about it of the particles from without would, owing to their distance from it, surpass that of those within and a direct rotation result.

Laws of force graphically shown.

The attraction, and thence the velocities in the different parts of the nebula, may be well shown graphically.

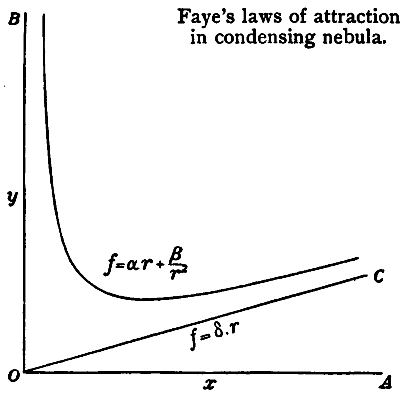


Fig. XVIII.

Faye's equation holds only when α and β are functions of r as well as of t. It, therefore, fails to give a good representation of what occurs throughout at a given moment. Furthermore, the equations do not hold up to the axis of y, as a discontinuity occurs so soon as we enter the central mass.

A better picture is the following, somewhat changed from Ligondès. As the matter gets drawn into the central mass, the attraction at the outer parts of the original nebula grows less and less, therefore C sinks to F, and the successive curves of the attraction become OC, DD, EE, FF.

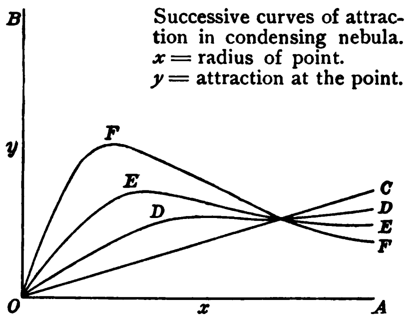


Fig. XIX.

The velocities at different distances follow a similar law.

This shows, as Ligondès points out, that there is a maximum velocity somewhere in the centre of the nebula, which degrades on both sides, so that we should have a plan of velocities for outside and inside portions of the nebula, thus:—

Effect on rotation.

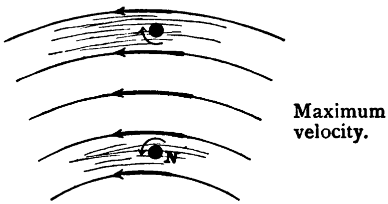


FIG. XX.

Supposing the density either the same throughout or to increase toward the centre, we should have, if the various planets were formed simultaneously, a retrograde rotation for the outer, a direct rotation for the inner ones.

New congruities since Laplace.

In addition to the ten congruities known in the time of Laplace, we must now add others from knowledge acquired since, to wit:—

XI. All the satellites turn the same face to their primaries (so far as we can judge).

XII. Mercury and probably Venus do the same to the Sun.

XIII. One law governs position and size in the solar system, and in all the satellite systems.

XIV. Orbital inclinations in the satellite systems increase with distance from the primary.

XV. The outer planets show a greater tilt of axis to orbit-plane with increased distance from the Sun (so far as detectable).

XVI. The inner planets show a similar relation.

Tidal friction in partial explanation.

Tidal friction explains xi. and xii.; xiii., xiv., xv., and xvi. are as yet unexplained.

Tidal friction fails with axial inclinations.

Tidal friction would account for xiv., but only on the supposition that the outer satellites were given off first. This is contrary to Faye's theory, largely so to Ligondés's, and is not championed by any other, for Laplace's supposition with regard to this point cannot stand.

Not only must the outer satellite have been given off the first, but very long before the next inner one, and so on for all; for tidal friction is potent as the inverse sixth power of the distance.

A similar objection holds against the attempt to explain the increased tilt of rotation axis to orbit planes, as distance from the Sun increases — both for the outer and the inner planets. This increased tilt with increased distance is well worth particular notice. It may be seen in the following table.

Planet.	*Inclination of Equator to Orbit-plane.*
Neptune	145°(?)
Uranus	98°(?)
Saturn	27°
Jupiter	3°
Mars	25°
Earth	23½°
Venus	0°(?)
Mercury	0°

We cannot be certain of Uranus and Neptune because we cannot see their surfaces well enough to be sure of the position of their axes, but the planes in which their satellites revolve makes the value given altogether likely.

The tidal friction explanation of this would make Neptune very much the oldest planet, Uranus very much the next so, and so on. But the explanation is not satisfactory.

Small amount of momentum of solar system.

Our solar system has, as I have said, a very small relative moment of momentum; only the one thousandth part of what it might have as exemplified in the system of α Centauri.

Explicable by collision of two suns.

One supposition will account for the small moment of momentum of the system, without supposing the individual motions so nearly balanced at the start. The moment of momentum would be small if the principal mass were initially collected in the centre of the nebula. Now this would be the case if the present system had been formed by the collision of two bodies. For, when dealing with such masses, the elasticity may be considered small, and, in default of elasticity, the matter after the collision would be found chiefly near the scene of the catastrophe if the impact were in the line joining their centres. The collision in space of two bodies happening head on is, of course, one of which the chances are very small, and, were it not for another fact, might be dismissed from reasonable consideration.

Physical condition of meteorites sustains this idea.

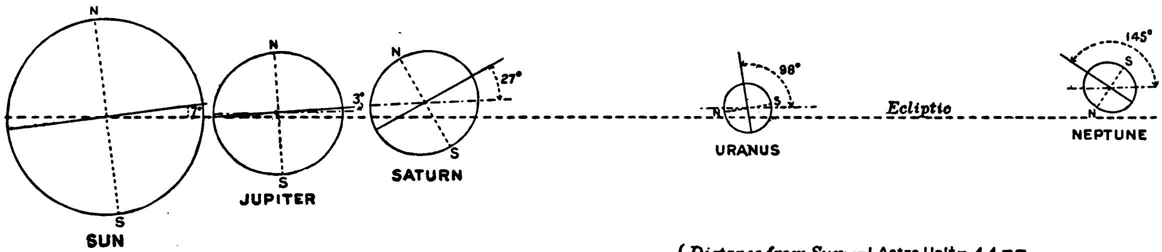
This fact is the present constitution of the unattached particles of the system, the meteorites. As we saw in a preceding lecture, these fragments betray a previous habitat. Their character shows that they came from the interior of a great cooled mass which once had been intensely heated. They are therefore proof of the prior existence of a great sun, and that they should be now strewn in space makes the theory of a subsequent collision far less improbable.

Distribution of density after collision.

If such a collision occurred, the fragments would be scattered more sparsely according to their distance from the scene of the catastrophe, and we may perhaps assume the law governing this sparseness to be the curve of probability,

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}.$$

AXIS INCLINATIONS OF THE MAJOR PLANETS



SCALE {
Distance from Sun—1 Astro.Unit = 4.4 mm.
Diameters of Planets–5200 miles = 1 mm.
Diameter of Sun----33120 miles = 1 mm.

FIG. XXI.

Then the probable amount of matter lying between x and $x+dx$ is $\frac{h}{\sqrt{\pi}}e^{-h^2x^2}dx$, and considering x to be y, and y, x, we have similarly for the probable amount of matter lying between y and $y+dy$, $\frac{h}{\sqrt{\pi}}e^{-h^2y^2}dy$.

The probable amount, therefore, in the rectangle $dx\,dy$ is $\frac{h^2}{\pi}e^{-h^2(x^2+y^2)}dx\,dy = \frac{h^2}{\pi}e^{-h^2r^2}a$, where $a = dx\,dy$, and r denotes its distance from the origin, or, in this case, the centre of the Sun.

For the amount in a ring at distance r, we have $a = r\,dr$.

Effect on rotation.

Consequently it is evident that there is less relative variation in the density with the distance as one goes out. *A fortiori*, therefore, when the planetary masses do not increase in like proportion, the two ends, the outer and the inner, of the strip or bunch of matter that went to make each up, vary less in density *inter se*. In the resultant rotation, the speed of the separate particles counts for more, relatively, than their density, and, in consequence, for the outer planets we should get a retrograde rotation; for the inner, a direct one.

Inner planets later formed.

That the inner planets were not formed early in the system's development seems pointed at pretty conclusively by their several masses. Present mechanical conditions of the matter inside Jupiter's orbit appear to point to the pre-existent influence of Jupiter upon it before birth. Not only do the amounts of matter in the several terrestrial planets indicate this, but the lack of formation of a planet in the gap occupied by the asteroids seems well-nigh conclusive on the point.

Shown by axial rotation.

A glance at the axial inclinations of the outer and the inner planets betrays a break in the symmetry of their arrangement. Each, taken by itself, evinces a gradual righting of the axis as one approaches the Sun. This appears strikingly from the table of the inclinations of the equators of the several planets to the planes of their orbits.

Jupiter's action the cause.

This, too, seems to point to the action of Jupiter. On the whole it appears probable that Jupiter existed before any of the small planets within its orbit, and profoundly modified them prenatally.

Conclusion.

We thus come to a conclusion in which nothing is concluded: but we need not regret that. The subject becomes the more exciting for remaining yet a mystery. We now know of relations so systematic and singular that we are sure some law underlies them, and it is rather pleasant than otherwise to have that law baffle our first attempts at discovery.

Future of the system.

But though we cannot as yet review with the mind's eye our past, we can, to an extent, foresee our future. We can with scientific confidence look forward to a time when each of the bodies composing the solar system shall turn an unchanging face in perpetuity to the Sun. Each will then have reached the end of its evolution, set in the unchanging stare of death.

Then the Sun itself will go out, becoming a cold and lifeless mass; and the solar system will circle unseen, ghostlike, in space, awaiting only the resurrection of another cosmic catastrophe.

The Riverside Press
Electrotyped and printed by H. O. Houghton & Co.
Cambridge, Mass., U. S. A.